Patterns and Scaling Properties in a Ballistic Deposition Model

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We study a ballistic deposition model in 1+1 dimensions in which the incident angles (the angles between the incident trajectories and the substrate) of incoming particles are randomly distributed in the range $[\theta, \pi - \theta]$. We find a sharp morphological transition at a critical angle $\theta_c \approx 10°$. For $\theta > \theta_c$, the scaling properties of the interface are described by the Kardar-Parisi-Zhang equation. For $\theta < \theta_c$, the shadowing effect leads to a very different morphology. We determine the scaling properties of this new universality class numerically and analytically.

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Growth and form have been a subject of interest for a long time [1]. Much progress has been made recently in understanding how patterns emerge under nonequilibrium growth conditions. A well-studied example is the ballistic deposition model [2] which, among other things, models the growth in sputter and vapor depositions. The growth rule of the model is very simple: particles rain down vertically to a d-dimensional substrate and stick to the aggregate or the substrate upon first contact. Such a simple model gives rise to a rather interesting structure —while the aggregate is compact [3] the growing surface of the aggregate is a self-affine fractal and its scaling properties are well described by the Kardar-Parisi-Zhang (KPZ) equation [4]. Models with fixed incident angles other than 90° were also studied and were found to produce columnar structures [5]. On the other hand, some experiments on sputtering showed a very different morphology [6]. The most striking feature of the morphology is that the surface structures have an obvious length scale which grows with the film thickness [6]. This "coarsening" is believed to be due to the nonlocal shadowing of the incoming particle flux by the surface structures; i.e., bigger structures shadow smaller ones and hence grow faster. Karunasiri, Bruinsma, and Rudnick [7] first studied the effect of shadowing by a simple "grass model" and showed that nonlocal shadowing leads to a growth instability. Several authors also studied the shadowing effect by various models [8–10]. Despite the extensive studies on the ballistic deposition model and studies on various "shadow" models, the relationship between the growth condition and the morphology in sputtering is still not well understood.

In this Letter, we study a ballistic deposition model in which the incoming particles can have a range of incident angles. The model retains the simplicity of the unidirectional ballistic deposition model and yet is more realistic for sputter and vapor deposition. We shall see that this model has some very interesting behaviors. In particular, as the range of the incident angles is varied, there is a sharp transition in morphology. For not very large ranges of incident angles, the scaling properties of the interface are described by the KPZ equation. At very large ranges of incident angles, shadowing leads to nonlocal competition for incoming flux between different branches of the aggregate. The competition on all length and time scales gives rise to a scale-invariant morphology characterized by the coarsening of an apparent length scale. The morphology is similar to that seen in the experiments of Ref. [6] and is in a different universality class than the one given by the KPZ equation.

The growth rule of the model in 1+1 dimensions is the following. Start with a substrate line of length L; particles (disks of diameter unity) are released from a line source of the same length L which is far above and parallel to the substrate. The particles move towards the substrate ballistically with incident angles (the angle between the particle trajectory and the substrate) randomly distributed in $[\theta, \pi - \theta]$, and the particles stick to the growing aggregate or the substrate at first contact. We use a periodic boundary condition so that a particle leaving the left edge comes in from the right edge at the same height and with the same incident angle. Our simulations were carried out off lattice to eliminate any lattice effect.

In Fig. 1, we show a series of snapshots of a growing aggregate in the case of $\theta = 0$ in which the shadowing effect is the largest. We can clearly see that initially many treelike branches compete with one another and that the bigger ones win the competition. The bigger branches then compete among themselves and so on. For a finite substrate, eventually there will be only one branch left growing. We note that almost all the branches, including the dominating one, are stable to the tip splitting. We also note that there is a characteristic wedge angle at the tips of the large branches. Define the width of the interface to be $w^2 = \langle [y(x,t) - \langle y(x,t) \rangle]^2 \rangle$, where $y(x,t)$ is the maximum height of the aggregate at position x (ignoring overhangs), t is the number of particles in the aggregate, and $\langle \cdot \rangle$ denotes spatial average [11]. In a finite substrate and for very large t, there is only one branch with the wedge angle. If we ignore the deep groves, the width of the tip of the branch is simply $w \sim L^{\zeta}$, with

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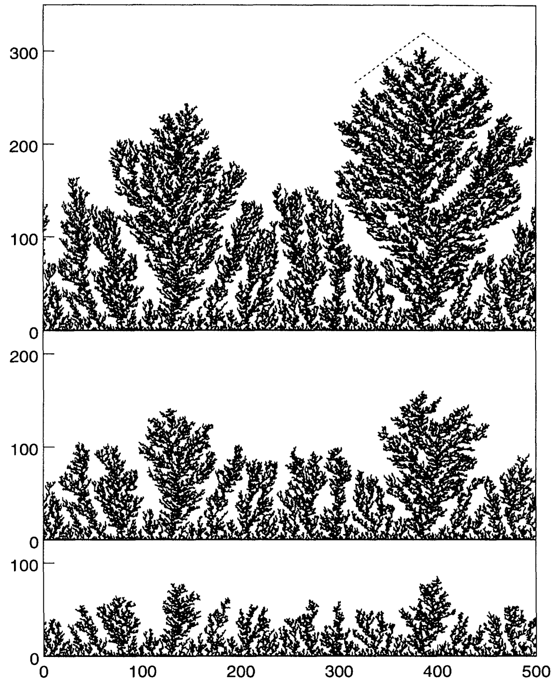

FIG. 1. Snapshots of a growing aggregate for $L = 500$ and $\theta = 0$. The shadowing effect is apparent. The smaller clusters are shadowed by the larger ones and hence grow slower. The number of particles in the aggregates are, from bottom to top, 10 000, 20 000, and 40 000. Note the wedge angles in the big clusters. The dotted lines in the uppermost figure indicate the wedge angle of 104.4° as given by Eq. (1).

$\zeta = 1$. We measured ζ in the simulation and we found $\zeta = 1.0 \pm 0.1$. For small t, w increases with t. In Fig. 2, we show the log-log plot of w vs t in the case of $\theta = 5°$. If we write $w \sim t^{\beta}$, then $\beta \approx 0.7$ and seems to increase with t. The scaling arguments presented later in the paper would suggest that $\beta = 1$ for small θ. It may well be that there is some kind of finite size or crossover effect. For the morphology like Fig. 1, a more natural quantity to measure is, perhaps, $n(s,t)$—the number of branches per unit length with s particles at time t [12]. We found that $n(s,t) \sim s^{-\tau} f(s^{\sigma}/t)$ with $\tau = 1.47 \pm 0.05$ and $\sigma = 0.53 \pm 0.05$ (Fig. 3). The identity $\int sLn(s,t)ds = t$ implies that $\tau + \sigma = 2$ [12], and our data are consistent with this equality. For a nonzero but small θ, we found that the morphology is similar to Fig. 1 with a slightly different wedge angle and that all the scaling exponents (apart from the uncertainty for β) are the same as for $\theta = 0$. Thus for small θ the scalings are different from KPZ and we have a new universality class.

Naively, one may expect that the shadowing should not be important at large length scales. The KPZ scaling implies a "flat" interface. The tilt angle of the interface on length scale ℓ is w/ℓ, and there will be no shadowing on length scales larger than ℓ if $w/\ell < \tan\theta$. In KPZ scaling,

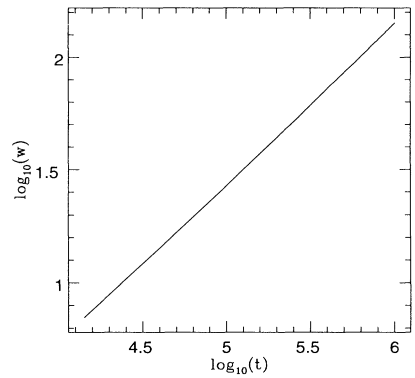

FIG. 2. The width w of the interface as a function of t for $L = 2000$ and $\theta = 5°$.

$w \sim \ell^{1/2}$ for $t >> \ell^{3/2}$ [4], giving $w/\ell \sim 1/\ell^{1/2}$. So there will be no shadowing on large enough length scales after long enough times if the interface is governed by the KPZ scaling. However, the KPZ scaling has to build up "locally" (the KPZ equation is a local equation). Therefore, if there are some "local" structures which scale differently from and propagate faster than KPZ, a different universality class could emerge. We think that the stable wedge angles in Fig. 1 play such a role.

To understand the wedge angle one would need a complete knowledge of the growth velocity, which we do not have at present. We make the plausible assumption that the local normal growth velocity at a point on the interface is proportional to the exposure angle within which it can receive the incoming particle flux [7]. Let us first consider the case of $\theta = 0$. If we ignore the shadowing from other branches, the normal velocity of one side of

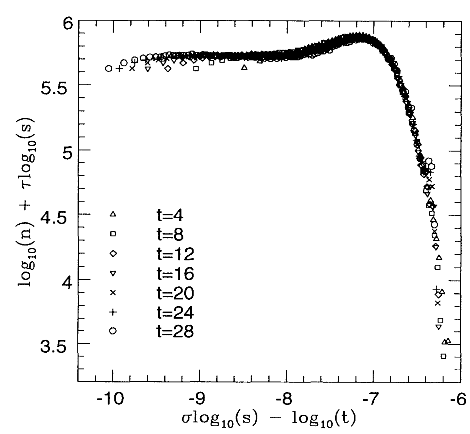

FIG. 3. Cluster size distributions at different times for $L = 9898$ and $\theta = 0$. The time t in the figure (number of particles in the aggregate) is measured in the unit of 14 000. $\tau = 1.47$ and $\sigma = 0.53$ are used in the figure to scale the curves of different times.

the wedge is simply $v_n = \pi - \alpha$, where α is the angle between the wedge side and the substrate (so the wedge angle is $\pi - 2\alpha$). The velocity in the y direction is then $v_y = v_n/\cos\alpha = (\pi - \alpha)/\cos\alpha$. Equating v_y on the wedge side with the velocity of the tip which is π, we have the equation for steady state solutions:

$$\frac{\pi - \alpha}{\cos\alpha} = \pi. \tag{1}$$

There are two solutions for Eq. (1): $\alpha = 0$ and $\alpha = \alpha^* \approx 37.8°$. It is easy to see that the first solution is unstable and the second one is globally stable. The later solution corresponds to a wedge angle of about 104°, which is in very good agreement with the simulations (Fig. 1). Encouraged by the success, we extend the analysis to the case of $\theta \neq 0$. In this case, $v_y = [\pi - \theta - \max(\alpha,\theta)]/\cos\alpha$ on the side of the wedge and the velocity at the tip is $\pi - 2\theta$. The steady state equation is

$$\frac{\pi - \theta - \max(\alpha,\theta)}{\cos\alpha} = \pi - 2\theta. \tag{2}$$

For small θ there are three solutions for Eq. (2): $\alpha = 0$, $\alpha_1 \approx \theta + (\pi/2)\theta^2$, and $\alpha_2 \approx \alpha^* - \theta(2\cos\theta^* - 1)/(\pi\sin\theta^* - 1) \approx \alpha^* - 0.63\theta$, as depicted in Fig. 4(a). It is easy to see that the solution α_2 is stable and the solution α_1 is unstable. The solution $\alpha = 0$ is marginally stable; i.e., $dv_y/d\alpha|_{\alpha=0} = 0$ and $d^2v_y/d\alpha^2|_{\alpha=0} > 0$. Therefore, for small θ, the aggregate has a similar morphology as that of Fig. 1 with a slightly different wedge angle. We have done simulations with small θ and found that the observed wedge angles are consistent with the values given by the stable solution of Eq. (2). The most interesting aspect of Eq. (2) is that as θ increases it goes through a bifurcation. Namely, as θ increases the two roots α_1 and α_2 move toward each other and at $\theta = \theta_c \approx 10.4°$ they merge into one root $\alpha = \alpha_c \approx 21.1°$, corresponding to a wedge angle of about 138° [Fig 4(b)]. For $\theta > \theta_c$, $\alpha = 0$ is the only solution for Eq. (2) and there will be no wedge angles at the interface [Fig. 4(c)]. The growth velocity close to $\alpha = 0$ depends on α quadratically—the same as that in the KPZ equation. Thus in this case, we expect that the interface would be "flat" and its scaling properties would be in the universality class of KPZ. In Fig. 5 we show a series of snapshots of the aggregate for $\theta = 15°$ which is slightly above θ_c. Indeed, there are no apparent wedge angles and the interface is very similar to that of the original unidirectional ballistic deposition model. We have measured the width w of the interface and it scales like $w \approx t^\beta$ at early times and $w \approx L^\zeta$ at large times, with $\beta = 0.34 \pm 0.05$ and $\zeta = 0.5 \pm 0.02$, confirming that it is in the same universality class as the KPZ equation.

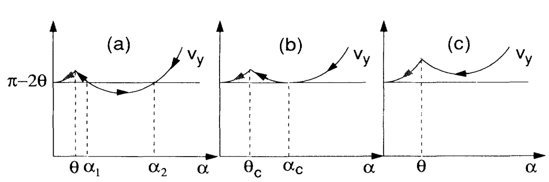

FIG. 4. The sketch of the left and right hand sides of Eq. (2) as functions of α. (a) $\theta < \theta_c$; (b) $\theta = \theta_c$; and (c) $\theta > \theta_c$.

We now proceed to calculate the cluster-size distribution of the new morphology ($\theta < \theta_c$). For simplicity we consider the case of $\theta = 0$. Let us examine the competition between two clusters which are distance ℓ apart. We ignore the presence of all other clusters at this point. If one cluster is higher than the other by the amount of δh, its tip will receive more particle flux by the amount of $\tan^{-1}(\delta h/\ell) \approx \delta h/\ell$, for $\delta h \ll \ell$. The equation governing the evolution of δh is then [7,10]

$$\frac{d\delta h}{dt} \approx \frac{\delta h}{\ell}. \tag{3}$$

So, $\delta h \approx \epsilon \exp(t/\ell)$, where ϵ is the initial height difference of the two competing clusters. We see that the initially slightly higher cluster will "take over" the other in a time scale of the order of $t \sim \ell$. Thus there is a "hierarchical" picture of competition—nearby clusters compete in early times and the winners which are further apart will compete in later times, and so on. We can see this in Fig. 1. Since the time or the height when clusters of distance ℓ apart compete with each other is proportional to ℓ, the number of clusters with heights larger than h,

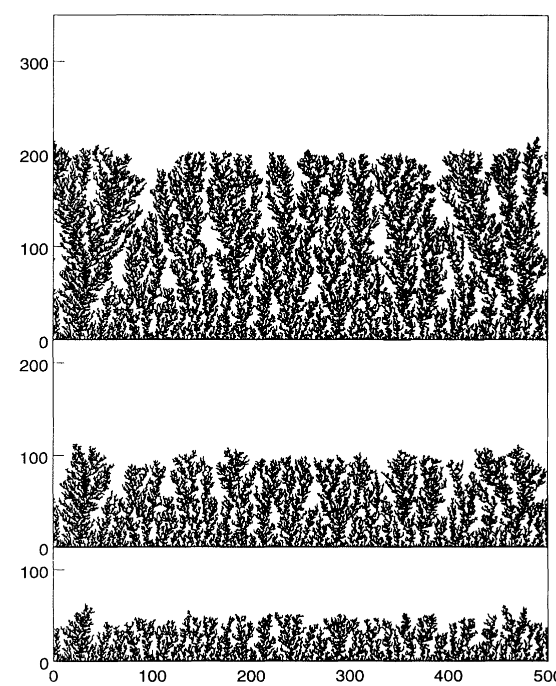

FIG. 5. Same as Fig. 1, except that $\theta = 15°$.

$N(h)$, is proportional to h^{-1}. We found numerically that the horizontal width of a cluster is proportional to its height, which implies that the number of particles in the cluster $s \sim h^2$. Combining this with $N(h) \sim h^{-1}$, we get $N(s) \sim s^{-0.5}$ which gives $n(s) \sim s^{-1.5}$, in good agreement with the simulation results (Fig. 3).

In conclusion, we have studied the ballistic deposition model in which incoming particles come from a range of angles at random. As the range of incident angles is increased, we found a sharp morphological transition from the KPZ growth to a new regime with morphology characterized by stable wedges [13]. We have developed a simple equation for the wedge angle. The morphological transition is due to a bifurcation in the equation. The scaling properties in the new morphology are different from the KPZ universality class. There is an apparent coarsening process in the morphology, similar to what has been observed in some sputtering experiments. This coarsening can be characterized by the cluster-size distribution function $n(s,t)$ whose exponents can be calculated analytically by considering competitions of different clusters due to shadowing.

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[13] A morphological transition from rough surface to columnar structure was found in a solid-on-solid model in Ref. [9]. There is a temperature in the model which plays a crucial role in the transition. It seems that the mechanism for the transition in our model is very different.