Dynamics and Noise Spectra of a Driven Single Flux Line in Superconductors

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We study the low temperature dynamics of a single flux line in a bulk type-II superconductor, driven by the Lorentz force acting near the sample surfaces, both near and above the depinning threshold. We find a novel instability of the flux line motion at large driving currents. The power spectrum of the voltage noise generated by the moving flux line in the presence of random pinning has a $\omega^{0.5}$ and $\omega^{-0.5}$ form for $\omega < \omega_l$ and $\omega > \omega_l$, respectively, where $\omega_l \to 0$ at the depinning threshold.

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Equilibrium and dynamic properties of flux lines in type-II superconductors have fascinated condensed matter physicists ever since their postulated existence by Abrikosov [1]. The need to understand the new high temperature (type-II) superconductors for both fundamental and applicational purposes provided new impetus for studying the physical properties of flux lines. Many recent studies have concentrated on the statistical mechanics of a large collection of interacting flux lines [2], in particular related to the questions of the type of novel phases they form in a high T_c superconductor [3,4], collective motions of flux lattices [5,6], and the dynamics of the nonequilibrium Bean critical state [7–10].

In this Letter, we focus instead on the dynamical properties of a driven *single* flux line in a bulk type-II superconductor, be it a high T_c type or a conventional type, at low temperatures. Physically, this corresponds to when H is only slightly larger than H_{c1} such that the flux lines density is so small that their motion can be regarded as independent of one another. The dynamics of a single flux line represents a well defined problem which we study here in detail, so as to build a sound foundation for understanding the complex physics of the interacting flux lines. All recent theoretical works on driven interfaces in random media have assumed a bulk driving force [11,12]. An important new feature in our model is the realization that the Lorentz force acts on a flux line *at the sample surfaces* [13], rather than uniformly from inside the bulk of the sample, since the applied current is confined to within a thickness λ from the surfaces ($\lambda \sim 0.1$ μm is the London penetration length). We shall see that this new realization implies an interesting *instability* for a driven single flux line at high driving Lorentz force (high current density j). On the other side, at small applied current the flux line is "pinned" by random impurities, such that its average velocity $\bar{v}=0$. At a critical current density j_c, which depends on the density and the strength of the pinning centers in a given sample and on the sample width in the direction of the H field, the flux line starts to move, i.e., $\bar{v}>0$. We shall show that the critical properties of this "depinning" transition give rise to interesting noise behavior for the induced voltage due to flux motion.

For simplicity, we model the dynamics of a single flux line as a two dimensional problem, defined by its shape function $y(x,t)$, with the y direction being the direction of the driving Lorentz force. We first derive the equation of motion for the single flux line. The basic equation is the overdamped Newton's law,

$$\gamma v_n(x,y(x,t)) = F_n(x,y(x,t)), \qquad (1)$$

where v_n is the normal velocity of the flux line at position $(x,y(x,t))$, F_n is the normal force per unit length, and γ is the damping coefficient ($\gamma \approx \phi_0^2/2\pi\xi^2c^2\rho_n$ from the Bardeen-Stephen model [14], with ϕ_0 the flux quantum, ξ the coherence length, and ρ_n the normal state resistivity). To obtain F_n, consider the energy of the flux line,

$$\varepsilon = \int dl[\sigma + U(x,y(x,t))] = \int dx\left[1+\left(\frac{\partial y}{\partial x}\right)^2\right]^{1/2}[\sigma + U(x,y(x,t))], \qquad (2)$$

where $\sigma \approx (H_c^2/8\pi)4\pi\xi^2\ln(\kappa)$, with H_c the critical field and $\kappa \equiv \lambda/\xi$ the Ginzburg-Landau parameter, is the flux line tension and $U(x,y)$ is the random part of the line energy arising due to impurities, defects, etc., with $\langle U(x,y)\rangle = 0$. In later calculations we assume that $U(x,y)$ is short-range correlated in space, so that $\langle U(x,y)U(x',y')\rangle = 2D\delta(x-x')\delta(y-y')$. The change of the flux line energy due to an infinitesimal displacement of the line is

$$\Delta\varepsilon = -\int dl\, F_n(x,y(x,t))\Delta n(x,y(x,t)) = -\int dx\, F_n(x,y(x,t))\Delta y(x), \qquad (3)$$

where Δn is the normal displacement and Δy is the displacement in the y direction, and they are related by $\Delta y = \Delta n\sqrt{1+(\partial y/\partial x)^2}$. Since in general $\Delta\varepsilon = \int dx[\delta\varepsilon/\delta y(x)]\Delta y(x)$, we get

$$F_n = -\frac{\delta\varepsilon}{\delta y(x)}. \qquad (4)$$

Combining Eqs. (1), (2), and (4) and after some algebra,

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we obtain the equation of motion for the flux line

$$\gamma v_n(x,y(x,t)) = [\sigma + U(x,y(x,t))]K(x,y(x,t)) - \hat{\mathbf{n}}\cdot\nabla U(x,y(x,t)), \quad (5)$$

where $K(x,y(x,t)) = \partial\sin\theta/\partial x$, with $\tan\theta = \partial y/\partial x$, is the local curvature and $\hat{\mathbf{n}} = (-\sin\theta,\cos\theta)$ the local unit normal vector of the flux line. Note that Eq. (5) is rotationally invariant and can be easily generalized to the case of a bulk driving force. Using $\partial y/\partial t = v_n\sqrt{1+(\partial y/\partial x)^2}$, Eq. (5) can be rewritten in the x-y coordinates as

$$\gamma\frac{\partial y}{\partial t} = \frac{\sigma+U}{1+(\partial y/\partial x)^2}\frac{\partial^2 y}{\partial x^2} + \frac{\partial U}{\partial x}\frac{\partial y}{\partial x} - \frac{\partial U}{\partial y}. \quad (6)$$

At the boundary $x = \pm L/2$, a driving (Lorentz) force F in the y direction is applied at the two ends of the flux line. [This corresponds to a driving current I in the third (z) direction.] This driving force is balanced by the local line tension, so

$$F = \pm[\sigma + U(\pm L/2, y(\pm L/2,t))]\sin\theta(x = \pm L/2).$$

This gives the boundary condition

$$\partial y(x=\pm L/2)/\partial x = \pm F/\sqrt{(\sigma+U)^2 - F^2}.$$

As the current density is confined to a surface layer with thickness $\lambda \ll L$, the total Lorentz force can be estimated as $F \approx \phi_0 I/cD$, with D being the same width in the y direction. We ignore thermal fluctuations, corresponding to the low temperature regime where the flux line dynamics is dominated by the driving Lorentz force and the random pinning force.

Let us analyze the above flux line equation in the absence of randomness ($U(x,y)=0$). Physically this is a valid approximation when the driving force is very large ($j \gg j_c$) and the string moves with a large velocity. The equation of motion becomes

$$\gamma\frac{\partial y}{\partial t} = \frac{\sigma}{1+(\partial y/\partial x)^2}\frac{\partial^2 y}{\partial x^2} = \sigma\frac{\partial\theta}{\partial x}. \quad (7)$$

We look for the steady state solution

$$\sigma\frac{\partial\theta}{\partial x} = \gamma v = \text{const}. \quad (8)$$

Equation (8) can be integrated to give

$$y = -\frac{\sigma}{\gamma v}\ln\left[\cos\left(\frac{\gamma v}{\sigma}x\right)\right] + vt. \quad (9)$$

From the boundary condition $F = \pm\sigma\sin\theta(x=\pm L/2)$, we obtain the relation between the driving force F and the steady state velocity v:

$$v = \frac{2\sigma}{\gamma L}\sin^{-1}\frac{F}{\sigma}. \quad (10)$$

Note that the steady state velocity has a maximum value $v_{\max} = \pi\sigma/\gamma L$. An interesting aspect of the steady state solutions Eqs. (8)–(10) is the onset of an instability. As $F \to \sigma$, $v \to v_{\max}$ and the flux line contact angles with boundaries reach 0 [$\theta(x=\pm L/2) \to \pm\pi/2$]; i.e., the flux line "wets" the boundaries. For $F > \sigma$, there are no steady state solutions and the flux line will be stretched longer and longer. We can estimate the value of the driving current for this instability. As the current density is confined to a surface layer with thickness $\lambda \ll L$, the total Lorentz force can be estimated as $F \approx \phi_0 I/cW$, where I is the driving current and W the sample width in the y direction. Setting $F=\sigma$ gives the critical driving current (per width of sample) for the onset of the instability

$$\frac{I}{W} \approx \frac{c\phi_0\ln\kappa}{16\pi^2\lambda^2} \approx 10\ln\kappa\left(\frac{1000\,\text{Å}}{\lambda}\right)^2\frac{\text{A}}{\text{cm}}, \quad (11)$$

which will induce a field of the order of a few hundred G. This is within the experimental range for applying a driving current to a type-II superconductor. We emphasize that this instability should also occur in dense (interacting) flux line systems if the driving current is larger than that given by Eq. (11).

We now study the influence of random energy $U(x,y)$ on the motion of the flux line by numerically simulating Eq. (6). We coarse grain the system on the length scale of the pinning length l_c [2,15], below which the flux line is smooth. The x axis is then discretized into N meshes of size $\Delta x \sim l_c$. We use Δx as the unit for length and arbitrarily set $\Delta x = 0.01$. We have also set $\sigma = \gamma = 1$, so that both U and the driving force F are now measured in the unit of σ. We choose $U(x,y)$ to be superposition of Gaussian potentials of the form $u\exp\{-[(x-x_0)^2 + (y-y_0)^2]/w^2\}$, with u uniformly distributed in $[-u_0,u_0]$ and (x_0,y_0) randomly distributed in the x-y plane. The average number of potentials per area Δx^2 is n_p and we set the width of the potential $w = \Delta x/2$. The data presented in this paper are for the parameters $N = 10000$, $n_p = 0.44$, and $u_0 = 3\times10^{-4}$. For this set of parameters, we find a depinning threshold of $F_c \approx 0.055$. Because of

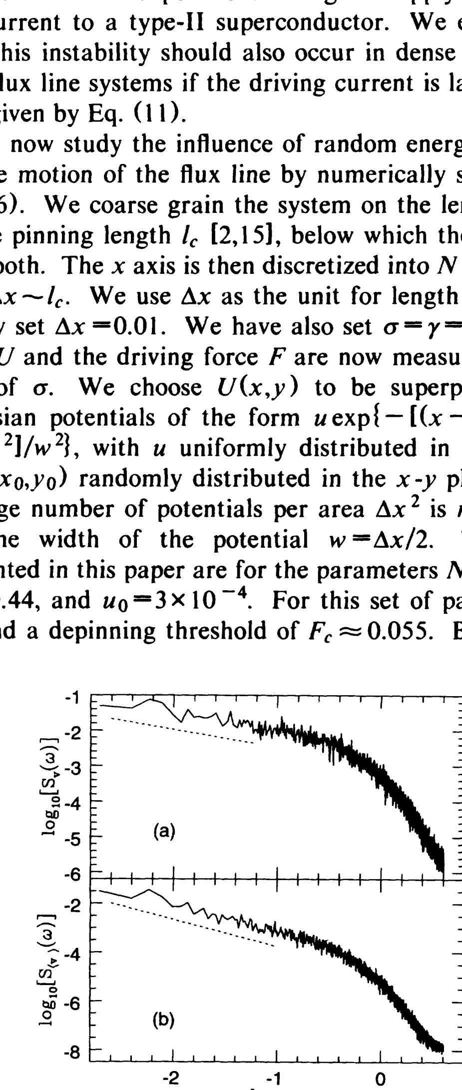

FIG. 1. Power spectrum for the velocity fluctuations at driving force $F=0.06$ which is just above the depinning threshold. (a) The spectrum for the velocities of the two end points, corresponding to measurable voltage fluctuations. The dotted line has slope -0.5. (b) The spectrum for the average velocity of the string. The dotted line has slope -1.1.

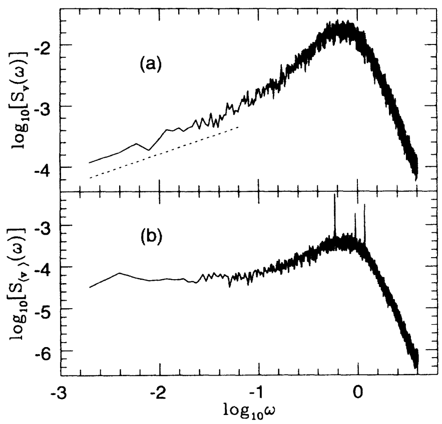

FIG. 2. Power spectrum for the velocity fluctuations at driving force $F=0.2$ which exceeds the depinning threshold F_c significantly. (a) The spectrum for the velocities of the two end points. The dotted line has slope 0.55. (b) The spectrum for the average velocity of the string.

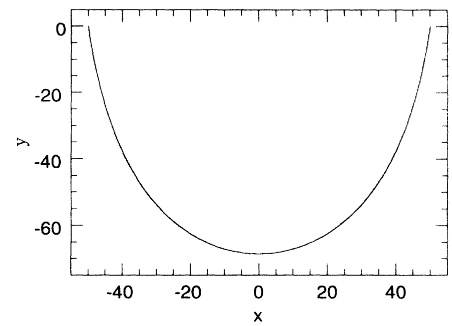

FIG. 3. Flux line profile in the case of $u_0=3\times10^{-4}$ and $F=0.99$. Also plotted is the curve given by Eq. (9) with $F=0.99$.

the random potentials U, the velocity of the string has interesting spatial and temporal fluctuations in the moving steady state. As the velocities of the ends of the string correspond to the induced voltage drops in the direction of the applied current, fluctuations in these velocities lead to measurable voltage noise. In Fig. 1(a), we plot the power spectrum of the temporal fluctuations of the end point velocity of a string in steady state, at a driving force F slightly larger than the depinning threshold F_c. We observe that the low frequency part of the spectrum has the form $S_v(\omega)\sim\omega^{-0.5\pm0.1}$. In Fig. 1(b), we plot the power spectrum for the fluctuations of the spatially averaged velocity $[\langle v\rangle(t)\equiv(1/N)\sum_i\Delta y_i/\Delta t]$, which shows also a scaling regime at low frequencies, $S_{\langle v\rangle}(\omega)\sim\omega^{-1.1\pm0.1}$. In Figs. 2(a) and 2(b), we plot the same power spectra for the string at a driving force significantly larger than the depinning threshold. The low frequency regime seen in Figs. 1(a) and 1(b) is now squeezed by a newly emerging low frequency regime with a *positive power*, $S_v(\omega)\sim\omega^{0.55\pm0.1}$ for the end velocity, and a white noise spectrum for the averaged velocity. For a very large driving force, the fluctuations due to the random U are insignificant (we will see in the next paragraph that the fluctuations are of order $1/v$); the shape and the velocity of the flux line are well approximated by Eqs. (9) and (10). In Fig. 3, we plot the instant shape of the flux line along with Eq. (9) for $F=0.99$ which is very close to the onset of the instability. The two curves are indistinguishable in the plot. The measured velocity of the flux line is $v=0.0284$, which is very close to the value $v=0.0286$ given by Eq. (10).

In order to understand qualitatively the above noise spectra, we turn to the fluctuations of the flux line around the average configuration $\langle y(x,t)\rangle$ which contains the trivial t dependence vt and a nontrivial x dependence due to the driving from the surface. To simplify the discussion, let us truncate Eq. (6) by keeping only terms linear in $y(x,t)$, and substitute $y(x,t)=\langle y(x,t)\rangle+\delta y(x,t)$ and $U(x,y(x,t))=\langle U(x,y(x,t))\rangle+\eta(x,y(x,t))$. Thus we get (with $\sigma+\langle U\rangle=\gamma=1$, for simplicity)

$$\frac{\partial\delta y(x,t)}{\partial t}=\frac{\partial^2\delta y(x,t)}{\partial x^2}+c_1(x)\eta(x,y(x,t))-\frac{\partial\eta(x,y(x,t))}{\partial y}+c_2(x)\frac{\partial\eta(x,y(x,t))}{\partial x}, \quad (12)$$

with $c_1(x)=\partial^2\langle y(x,t)\rangle/\partial x^2$ (the average of the string curvature at x) and $c_2(x)=\partial\langle y(x,t)\rangle/\partial x$ (the average of the string slope at x). Important features of Eq. (12) are (i) η is random in $(x,y(x,t))$ but not in (x,t) as in the case of thermal fluctuations; and (ii) due to a nontrivial x dependence of $\langle y(x,t)\rangle$, the model in (12) is not translationally invariant, which complicates the analysis of the problem. In a qualitative discussion, however, one can replace $c_1(x)$ by, say, $c_1(x=0)$, and $c_2(x)$ by $c_2(x=0)=0$ [note that $\langle y(x,t)\rangle$ is even in x]. Thus, we will ignore the last term in (12), and ignore the x dependence of $c_1(x)$ by replacing it with a constant, say, $c_1(x=0)$ or the spatially averaged curvature

$$\frac{1}{L}\int_{-L/2}^{L/2}c_1(x)dx=\frac{1}{L}[\partial\langle y(x=L/2,t)\rangle/\partial x-\partial\langle y(x=-L/2,t)\rangle/\partial x]\approx 2F/L\,.$$

[Note that at the depinning threshold, $F\to F_c\sim L$, the spatially averaged $c_1(x)$ has a *nonzero value*.] With these simplifications one gets

$$\frac{\partial\delta y(x,t)}{\partial t}=\frac{\partial^2\delta y(x,t)}{\partial x^2}+c_1\eta(x,y(x,t))-\frac{\partial\eta(x,y(x,t))}{\partial y}\,. \quad (12')$$

To proceed, we take η to be short-range correlated randomness, i.e., $\langle \eta(x,y)\eta(x',y')\rangle = 2D\delta(x-x')\delta(y-y')$. Equation (12′) has been discussed in some other contexts [15,16]. It contains a "random field" disorder, the term $c_1\eta$, and a "random bond" disorder, the term $\partial\eta/\partial y$. At large driving, we may approximate $\eta(x,y(x,t))$ by $\eta(x,vt)$, so that η can be approximated by uncorrelated noise $\eta(x,t)$, with $\langle \eta(x,t)\eta(x',t')\rangle = (2D/v)\delta(x-x')\times\delta(t-t')$. With this Eq. (12′) becomes easily tractable. So, for autocorrelations of velocity at some point x of the string we get $S_v(\omega) = \omega^2\langle|\delta y(\omega)|^2\rangle \sim \omega^{1/2}$ $(\omega \to 0)$, in agreement with our data at low frequencies in Fig. 2(a) [17]. For the spatially averaged velocity we get $S_{\langle v\rangle}(\omega) \sim \text{const}$ $(\omega \to 0)$, in agreement with our data in Fig. 2(b). We remark that this low-ω behavior is, for $c_1 \neq 0$, dominated by the "random field" term in (12′), whereas the "random bond" term yields only subdominant scaling corrections. For $c_1 = 0$, low-ω velocity noise is much weaker, of the form $S_v \sim \omega^{5/2}$. Recall, however, that $c_1 \neq 0$, and "random fields" dominate all the way down to the depinning transition and *at* the transition.

The above behavior holds in the range of the very low frequencies, $\omega < \omega_1$. As $F \to F_c$, $\omega_1 \to 0$, and the broad frequency range we observe for $\omega > \omega_1$, with $S_v(\omega) \sim \omega^{-0.5}$ and $S_{\langle v\rangle}(\omega) \sim \omega^{-1.1}$, must be a manifestation of the critical behavior of a *second order* depinning transition. Thus the correlation function obeys the scaling law

$$\langle \delta y(x,t)\delta y(x',t')\rangle \sim |t-t'|^{2\beta} f\left(\frac{|x-x'|}{|t-t'|^{1/z}}\right), \qquad (13)$$

for $|t-t'| < 1/\omega_1 \sim \bar{v}^{-1/(1-\beta)}$, where $\bar{v}$ is the time averaged velocity. For a (1+1)-dimensional random field system, Ref. [11] gives $\beta \approx \frac{3}{4}$ and $z \approx \frac{4}{3}$. Equation (13) implies that the autocorrelation of the velocity fluctuations $\delta v(x,t) = \partial\delta y(x,t)/\partial t$ decays as $|t-t'|^{-2(1-\beta)}$, so that $S_v(\omega) \sim \omega^{-(2\beta-1)} \sim \omega^{-0.5}$. This agrees with the low frequency part in Fig. 1(a). Equation (13) also implies that $S_{\langle v\rangle}(\omega) \sim \omega^{-(2\beta-1+1/z)} \sim \omega^{-1.25}$, in agreement with the low frequency part in Fig. 1(b). These scalings for $S_v(\omega)$ and $S_{\langle v\rangle}(\omega)$ hold above the frequency scale $\omega_1 \sim \bar{v}^{1/(1-\beta)} \sim \bar{v}^4$, which is strongly depressed to zero at the depinning transition where $\bar{v} \to 0$, as can be seen in Figs. 1(a) and 1(b).

In conclusion, we have studied the low temperature dynamics of a single flux line in a bulk type-II superconductor. The flux line is driven by the Lorentz force near the sample surface. There exists a critical driving current density j_c. For $j < j_c$ the flux line is pinned by impurities. For $j > j_c$ the power spectrum of the voltage fluctuations generated by the moving flux line has a form of $\omega^{0.5}$, for $\omega < \omega_1$, and $\omega^{-0.5}$, for $\omega > \omega_1$. The crossover frequency is $\omega_1 \sim \bar{v}^4$. The $\omega^{-0.5}$ part is related to the critical pinning-depinning transition which appears to be in the "random-field" universality class. The $\omega^{0.5}$ part is due to fluctuations on length scales longer than the correlation length of the transition [18]. At a large driving current, an instability sets in, in which the driving force generates new flux lines. This instability would change drastically the dissipation mechanism and the dissipation rate. Experimentally, it might be easier and even advantageous to study not a single line but a dilute concentration of flux lines. Sufficiently dilute flux lines will not interact with one another, and will lead to much larger measurable voltage signals, for both the average flux velocity as well as its temporal fluctuations.

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[17] The leading order nonlinear corrections to Eq. (12) contains a KPZ term $\sim(\delta y/\delta x)^2$ [M. Kardar, G. Parisi, and Y.-C. Zhang, Phys. Rev. Lett. **56**, 889 (1986)]. For the KPZ equation, one finds $S_v \sim \omega^{1/3}$. Among the probable reasons that we do not observe this scaling are the finite size effect and the fact that in our system the translational invariance is broken in the x direction by the surface driving force.

[18] This exponent 0.5 for low frequencies appears to be "super universal"; i.e., it is independent of the dimensions.