

# Earthquakes as a Self-Organized Critical Phenomenon

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The Gutenberg-Richter power law distribution for energy released at earthquakes can be understood as a consequence of the earth crust being in a self-organized critical state. A simple cellular automaton stick-slip type model yields  $D(E) \approx E^{-\tau}$  with  $\tau \approx 1.0$  and  $\tau \approx 1.35$  in two and three dimensions, respectively. The size of earthquakes is unpredictable since the evolution of an earthquake depends crucially on minor details of the crust.

## INTRODUCTION

The distribution of energy released during earthquakes has been found to obey the famous Gutenberg-Richter law [Gutenberg and Richter, 1956]. The law is based on the empirical observation that the number  $N$  of earthquakes of size greater than  $m$  is given by the relation

$$\log_{10} N = a - bm \quad (1)$$

The precise values of  $a$  and  $b$  depend on the location, but generally  $b$  is in the interval  $0.8 < b < 1.5$ . The energy released during the earthquake is believed to increase exponentially with the size of the earthquake,

$$\log_{10} E = c - dm \quad (2)$$

so the Gutenberg-Richter law is essentially a power law connecting the frequency distribution function with the energy release  $E$  (or other physical quantities such as the "seismic moment")

$$dN/dE \propto m^{-1-b/d} = m^{-\tau} \quad (3)$$

with  $1.25 < \tau < 1.5$ .

Despite the universality of the Gutenberg-Richter relation, there is essentially no understanding of the underlying mechanisms. It has been suggested that the power law is related to geometric features of the fault structure [Kagan and Knopoff, 1987], and indeed it has been pointed out by Mandelbrot [1982] that earthquakes occur on "fractal" self-similar sets. But, of course, this just shifts the problem to identifying the dynamical mechanism producing these geometric structures which are ultimately responsible for the earthquake dynamics.

In fact, power laws (and the lack of understanding of those) are quite common in nature. Recently, we have shown that dynamical systems may self-organize into a critical state similar to that of systems undergoing continuous phase transitions, with power law spatial and temporal correlation functions. In the following section we show that this behavior can be related fairly directly to earthquakes. Thus the Gutenberg-Richter law can be interpreted as a manifestation of the self-organized critical behavior of the earth dynamics. The fractal geometric distribution and the earthquake dynamics are the spatial and temporal signatures of the same phenomenon. Of course, any specific dynamical

model must necessarily be grossly simplified. The immediate goal is not to produce an accurate model but to point out a general mechanism leading to the power law distribution of earthquakes. In the following section an effort will be made to connect the concept of self-organized criticality to earthquakes.

## SELF-ORGANIZED CRITICALITY AND MODEL CALCULATIONS

It is generally assumed that the dynamics of earthquakes is due to a stick-slip mechanism involving sliding of the crust of the earth along faults [Stuart and Mavko, 1979; Sieh, 1978; Choi and Huberman, 1984]. When slip occurs at some location, the strain energy at that position is released, and the stress propagates to the near environment. While this picture is rather well established, no connection between stick-slip models and the actual spatial and temporal correlations has been demonstrated. It has been suggested that the stick-slip picture can be modeled as a branching process [Kagan and Knopoff, 1987]. The observed power law behavior is then rather remarkable since one would naively expect some exponential distribution, e.g.,  $D(E) \approx e^{-E/E_0}$ , where  $E_0$  is roughly the energy released at a single slip.

In simple dynamical systems with few degrees of freedom, and in extended equilibrium statistical systems, power laws are rare. One has to fine tune a parameter such as a dynamical coupling or temperature to arrive at a "critical point" in order to get power law correlations. But for dynamical systems in nature there is nobody to turn the knob, so where does the apparent criticality come from?

We have found that certain interacting dynamical systems naturally evolve into a statistically stationary state, which is also critical, with power law spatial and temporal correlations [Bak et al., 1987, 1988; Tang and Bak, 1988a, b]. It is essential that the systems are dissipative (energy is released) and that they are spatially extended with an "infinity" of degrees of freedom. Energy is fed into the system in a uniform way, either directly into the bulk or through the boundaries. The crust of the earth, subjected to the pressure from tectonic plate motion, may be viewed as a system of this kind.

At the stationary state there is a fragile balance between the local forces, adjusting the probability that a slip will propagate to a near neighbor precisely to unity. The probability of branching of the activity is compensated by the probability of "death" of the activity. The stationary state can be thought of as a critical chain reaction. Visually, the critical state can be thought of as the state of a steep sand

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heap which has been built from scratch by slowly adding particles. The avalanches caused by adding additional particles represent earthquakes. As the pressure builds up, the avalanches become bigger and bigger. At the critical state there is no characteristic time, space, or energy scale, and all spatial and temporal correlation functions are power laws. The power law size distribution is intimately related to the geometric self-similarity of active earthquake regions. The assumptions are that the system is large and that the driving force, i.e., the tectonic plate motion, is slow.

The models that we have studied are extremely simple "cellular automata" (see, for example, *Wolfram* [1985]). In principle, we could study three-dimensional partial differential equations, but the numerical calculations would be prohibitively time consuming, and we believe that the discretization does not affect the asymptotic long time and space behavior that we are interested in.

Consider a two-dimensional array of particles, for instance, on a square lattice  $0 \leq (i, j) \leq N$ , representing segments of a sliding surface. The particles are subjected to a force from their neighbors plus a constantly increasing "tectonic" driving force. When the total force on a particle exceeds a maximum local pinning force at the fault, the particle slips to a nearby position. Let the maximum pinning force be an integer  $Z_c$ . If at time  $t$  the system is in the state  $Z(i, j)$ , then the system at time  $t + \Delta t$  (where  $\Delta t$  is of the order of the distance between the locked elements divided by something like the speed of sound) is given by the rule

$$\begin{aligned} Z(i, j) &\rightarrow Z(i, j) - 4 \\ Z(i \pm 1, j) &\rightarrow Z(i \pm 1, j) + 1 \\ Z(i, j \pm 1) &\rightarrow Z(i, j \pm 1) + 1 \quad Z(i, j) > Z_c \end{aligned} \quad (4)$$

where the first equation simulates the release of strain (in proper reduced units) on the slipping particle and the subsequent equations represent the increase of force on the neighbor particles. Forces are conserved except at the boundaries; the macroscopic external forces are released only at the boundaries. The conservation of the propagating force may be appropriate for earthquakes but is not a general prerequisite for self-organized criticality.

The model is actually very close to the generally accepted "block spring" picture of earthquakes [*Burridge and Knopoff*, 1967; *Mikumo and Miyatake*, 1978, 1979]. This is precisely why we believe that our results apply to earthquakes; we do not have to invoke a new and different local mechanism.

Starting with a situation with no force,  $Z = 0$ , we simulate the increase in the driving force by letting

$$Z(i, j) \rightarrow Z(i, j) + 1 \quad (5)$$

at a random position  $(i, j)$ . One may think of a slow and uniformly increasing force. Since we are interested only in whether or not the force exceeds an integer critical value, it is enough to monitor the integer value of the force, which of course exhibits integer jumps like (5) only. The time scale of this process (a geological time scale) is assumed to be very large. This process is repeated until somewhere the force exceeds the pinning force  $Z_c$  and the rule (4) is applied: a unit energy is released. This may lead to instability at a neighbor position, in which case the rule (4) is applied to that position,

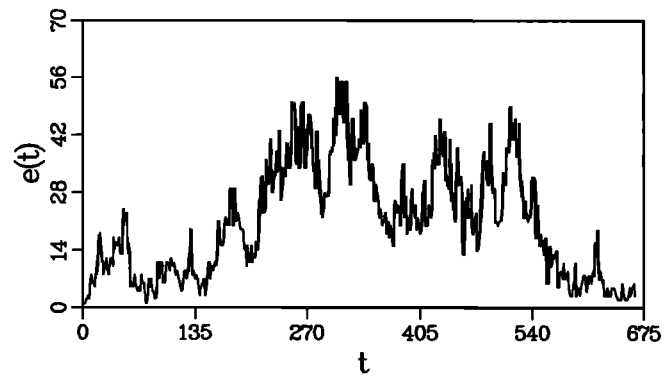


Fig. 1. Energy release versus time during a typical earthquake.

and so on. Eventually, the system will come to rest, namely when all  $Z$  values are less than  $Z_c$ . The total "domino" process initiated by (5) is the earthquake. Then (supposedly at a random much later time) the rule (5) is applied again, and so on. In the beginning there will be only small events, since  $Z$  values are generally small and a local slip is unlikely to propagate very far. But eventually, following rule (5), the average force  $\langle Z \rangle$  will reach a statistically stationary value which just allows the chain process to continue indefinitely. At that point there is no length scale and rule (5) may trigger earthquakes of all sizes limited only by the size of the system. This is the self-organized critical state.

Figure 1 shows the temporal evolution of the activity during a typical earthquake. Note the irregularity of the event. At several points the earthquake is almost dying, and its continued evolution depends on minor details of the crust of the earth far from the place of origin. Thus in order to predict the size of the earthquake, one must have extremely detailed knowledge on very minor features of the earth far from the place where the earthquake originated. If a mechanism of the type discussed here is indeed responsible for earthquakes, there is virtually no hope for ever making specific predictions. Perhaps the features at the beginning and the end can be thought of as foreshocks and aftershocks, respectively. *K. Ito and M. Matsuzaki* (Earthquakes as self-organized critical phenomena, submitted to *Journal of Geophysical Research*, 1989, hereinafter referred to as *IM*, 1989) have studied a slightly generalized version of our model in order to account for the Omori law for aftershock distribution.

The total number of segments which have slipped during the event is a measure of the total energy,  $E$ , released during the earthquake. Figure 2 shows the energy distribution at the stationary critical state. The distribution function indeed fits a power law  $D(E) \approx E^{-\tau}$  with  $\tau \approx 1$ . (The falloff at large  $E$  is a finite size effect.) Actually, it might be useful to think of the crust in the earthquake region as a three-dimensional medium developing ever-changing fault structures rather than considering a single fault. It is the crust as a whole rather than a single fault which is critical. The model can easily be generalized to three dimensions where one finds  $\tau \approx 1.35$  in even better agreement with observations.

Extensive numerical simulations in two and three dimensions have been carried out to further test the criticality [*Bak et al.*, 1987, 1988; *Tang and Bak*, 1988a, b]. In addition, there is now a substantial amount of analytical work, based mostly on renormalization group considerations, which

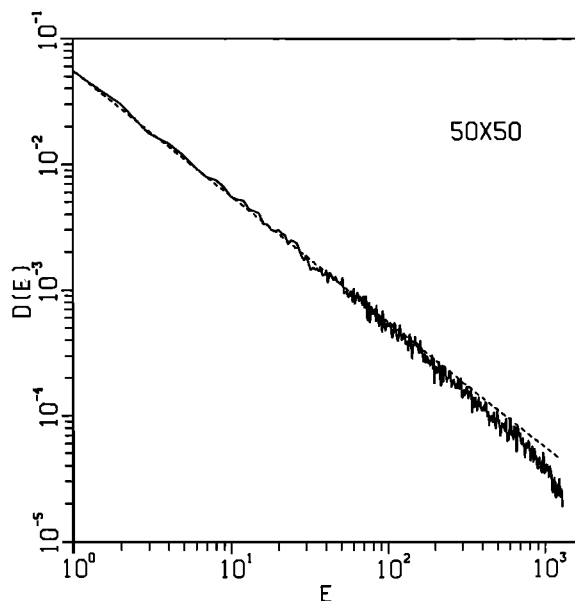


Fig. 2. Distribution of the total energy release  $E$ .

throws further light on the origin of the critical behavior and supports the conjecture of universality [Obukhov, 1989; Kadanoff *et al.*, 1989]. We refer the reader to this literature for further understanding of the mechanisms at work.

We have also studied models with random distribution of critical stress and random local connectivity of individual blocks. We find that this does not affect the exponents; this is essential for our mechanism to have any chance of success for a realistic system. Of course there is some ambiguity as to how to relate energies, areas, or "seismic moments" of an earthquake to the magnitude  $m$  of the earthquake, which is the quantity which is actually measured; this may change the estimate of the exponents but does not affect the main observation that the distribution is a power law. The powers may differ from different models, but there is also the distinct possibility, known from equilibrium critical phenomena as "universality," that the power depends only on geometric and topological features such as the spatial dimension. In this case the exponent derived from the model, however unrealistic, can be taken at face value for comparison with observations.

Since our model is essentially identical to previous stick-slip models, one might wonder why the criticality and the power law distributions have not been observed before. Mikumo and Miyatake [1978, 1979] studied the evolution of earthquakes from an initial random distribution of critical stresses but did not follow through to observe the emergence of the stationary critical state. Burridge and Knopoff [1967] studied a one-dimensional model and observed an exponential energy-frequency relation, implying a characteristic earthquake magnitude. Indeed, one-dimensional models are likely to decay exponentially because the low connectivity of the lattice prevents amplification of the activity [Bak *et al.*, 1987, 1988].

It has been suggested that earthquakes are a deterministic

"chaotic" phenomenon with few degrees of freedom. The criticality found here is of a fundamentally different nature since the infinity of degrees of freedom can not be reduced to a few. The unpredictability is caused by critical fluctuations rather than exponential sensitivity to initial conditions of a chaotic low-dimensional system. Dynamical phenomena with power law correlation functions are widespread in nature (weather, landscapes, biology, evolution of the universe [Mandelbrot, 1982]). We suggest that some of these can be viewed as "snapshots" of dynamical systems at the stationary critical state, although the specific modeling may be less straightforward than for earthquakes.

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#### REFERENCES

- Bak, P., C. Tang, and K. Wiesenfeld, Self-organized criticality: An explanation of  $1/f$  noise, *Phys. Rev. Lett.*, **59**, 381, 1987.
- Bak, P., C. Tang, and K. Wiesenfeld, Self-organized criticality, *Phys. Rev. A*, **38**, 364, 1988.
- Burridge, R., and L. Knopoff, Model and theoretical seismology, *Bull. Seismol. Soc. Am.*, **57**, 341, 1967.
- Choi, M. Y., and B. A. Huberman, A dynamical model for the stick-slip behavior of faults, *J. Phys. C*, **17**, L673, 1984.
- Gutenberg, B., and C. F. Richter, Magnitude and energy of earthquakes, *Ann. Geophys.*, **9**, 1, 1956.
- Kadanoff, L. P., S. R. Nagel, L. Wu, and S. Zhou, Scaling and universality in avalanches, *Phys. Rev. A*, **39**, 6524, 1989.
- Kagan, Y. Y., and L. Knopoff, Statistical short-term earthquake prediction, *Science*, **236**, 1563, 1987.
- Mandelbrot, B., *The Fractal Geometry of Nature*, W. H. Freeman, New York, 1982.
- Mikumo, T., and T. Miyatake, Dynamical rupture process on a three-dimensional fault with non-uniform frictions and near-field seismic waves, *Geophys. J. R. Astron. Soc.*, **54**, 417, 1978.
- Mikumo, T., and T. Miyatake, Earthquake sequences on a frictional fault model with non-uniform strengths and relaxation times, *Geophys. J. R. Astron. Soc.*, **59**, 497, 1979.
- Obukhov, S. P., The upper critical dimensional and  $\epsilon$ -expansion for self-organized critical phenomena, in *Random Fluctuations and Pattern Growth: Experiments and Models*, edited by H. E. Stanley and N. Ostrowsky, p. 336, Kluwer, Boston, 1989.
- Sieh, K. E., Prehistoric large earthquakes produced by slip on the San Andreas Fault at Pallett Creek, California, *J. Geophys. Res.*, **83**, 3907, 1978.
- Stuart, W. D., and G. M. Mavko, Earthquake instability on a strike-slip fault, *J. Geophys. Res.*, **84**, 2153, 1979.
- Tang, C., and P. Bak, Critical exponents and scaling relations for self-organized critical phenomena, *Phys. Rev. Lett.*, **60**, 2347, 1988a.
- Tang, C., and P. Bak, Mean field theory of self-organized critical phenomena, *J. Stat. Phys.*, **51**, 797, 1988b.
- Wolfram, S., Twenty problems in the theory of cellular automata, *Phys. Scr.*, **T9**, 170, 1985.

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