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Peak effect in superconductors: melting of Larkin domains

C. Tang[1], Xinsheng Ling[1](*), S. Bhattacharya[1] and P. M. Chaikin[1,2]

[1] *NEC Research Institute - 4 Independence Way, Princeton, NJ 08540, USA*

[2] *Department of Physics, Princeton University - Princeton, NJ 08544, USA*

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Abstract. – Motivated by the recent observations of the peak effect in high-T_c YBCO superconductors, we re-examine the origin of this unusual phenomenon. We propose that the sharp peak in the critical current as a function of temperature or magnetic field is an interesting manifestation of vortex-lattice melting in the presence of weak random pinning. Specifically, the rise of the critical current with increasing temperature or field is a result of a crossover from the Larkin pinning length to a length scale set by thermal fluctuations.

The properties of type-II superconductors are largely determined by the statics and dynamics of vortex lines. In particular, the critical current is directly related to the pinning of the vortex lattice by impurities and disorder. It was discovered more than 30 years ago by LeBlanc and Little [1] that the critical current in a type-II superconductor can increase with increasing temperature, or field, in a narrow range below the upper critical field $B_{c2}(T)$. The critical-current density has a pronounced peak below $B_{c2}(T)$ (fig. 1). This "peak-effect" phenomenon was found to be ubiquitous in conventional superconductors [2]-[4] and it has been observed recently in high-T_c superconducting YBCO crystals [5], [6]. Over the years, understanding the peak effect has been one of the most challenging tasks in the problem of vortex-lattice pinning [7]. Although the rise of critical current at the onset of the peak effect has been attributed to some kind of abrupt softening of the vortex lattice [8], [9], the underlying mechanism remains unknown. Various possible close connections between the peak effect and the vortex-lattice melting have been conjectured recently [10]-[12], [6]. In this paper, we propose a mechanism in which the peak effect is a manifestation of the vortex-lattice melting in realistic systems with quenched disorder. Specifically, we suggest that the rise of the critical-current with increasing temperature is a result of a crossover from the Larkin pinning length to the elastic length set by thermally excited free dislocations ([1]).

Let us first recall briefly the general features of the peak effect in both conventional and high-T_c YBCO superconductors. Figure 1 *a*) is a plot of the critical-current density as a function

(*) Present address: Department of Physics, Brown University, Providence, RI 02912, USA.
([1]) Part of this work was reported previously [13].

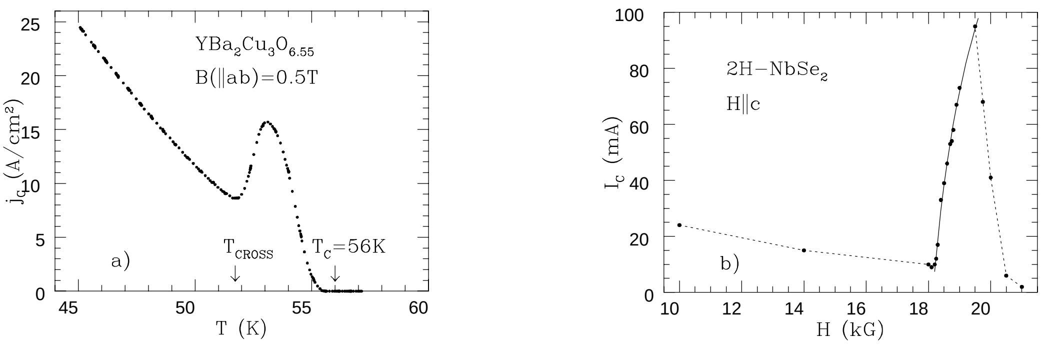

Fig. 1. – *a*) The temperature dependence of critical-current density for a YBCO crystal in a magnetic field (ref. [10]). We define T_{cross} as the onset temperature at which j_{c} starts to increase. *b*) I_{c} *vs.* H for 2H-NbSe$_2$ (ref. [9]). The solid line is a fit by eq. (2). The dotted lines are guide for the eyes.

of temperature extracted from ref. [5] for a YBCO crystal. With increasing temperature, j_{c} initially decreases monotonically, then suddenly rises, reaches a peak before finally dropping to zero. Similar behavior is also observed in low-T_{c} type-II superconductors, often as a function of the magnetic field [2]-[4]. Figure 1 *b*) is a reprint of fig. 1 (inset) of a recent work [4] on 2H-NbSe$_2$. It was found that the peak effect disappears when the sample is strongly disordered and has a high j_{c} [2], [4]-[6]; the critical current decreases monotonically to zero with increasing temperature or field.

The most striking aspect of the peak effect is the sharp rise of critical current with increasing temperature or field. According to the collective pinning theory of Larkin and Ovchinnikov (LO) [9], the critical current is determined by an elastic length: the pinning length. Larkin [14] showed that random pinning breaks the vortex lattice into domains of correlated regions within each of which the vortex lines interact elastically. The size of the Larkin domains can be estimated by a simple energy consideration. The vortex lattice deforms to take advantage of the random pinning potential at the cost of the elastic energy. The total unit volume energy change is [9]

$$\delta F = C_{66}\left(\frac{r_{\mathrm{p}}}{R}\right)^2 + C_{44}\left(\frac{r_{\mathrm{p}}}{L}\right)^2 - f r_{\mathrm{p}}\left(\frac{n}{V}\right)^{1/2}, \tag{1}$$

where C_{66} is the shear modulus of the lattice, C_{44} the tilt modulus, r_{p} the range of the pinning potential, f the typical force of an individual pin, n the pinning density, R and L are the transverse (to the field) and longitudinal (along the field) dimensions of the domain, and $V = R^2L$. The minimization of eq. (1) gives the pinning lengths R_{c} and L_{c}: $R_{\mathrm{c}} \sim C_{66}^{3/2}C_{44}^{1/2}r_{\mathrm{p}}^2/nf^2$, $L_{\mathrm{c}} = (C_{44}/C_{66})^{1/2}R_{\mathrm{c}}$. In very thin samples with a perpendicular field, if the pinning is so weak that L_{c} is greater than the sample thickness, the problem becomes two-dimensional (2D) and only $R_{\mathrm{c}} \sim C_{66}r_{\mathrm{p}}/n^{1/2}f$ is relevant. In the LO theory, the critical-current density is determined by equating the Lorentz force with the typical pinning force in a domain: $j_{\mathrm{c}}B = (nf^2/V_{\mathrm{c}})^{1/2}$. Thus, the rise in j_{c} can be accounted for if the volume of Larkin domain V_{c} drops faster than nf^2 in some field or temperature range. The central question here is what mechanism does that.

It was suggested [9] that the peak effect is due to the softening of the tilt modulus C_{44} as B approaches $B_{\mathrm{c2}}(T)$ [15]. This explanation of the peak effect has serious difficulties. First, it does not account for the temperature dependence that j_{c} rises with increasing T [11]. The second difficulty of this mechanism is that the peak effect has been observed in thin films [3] and in very thin $NbSe_2$ crystals with pinning weak enough such that L_c well exceeds the sample thickness [4] ([2]), in which C_{44} does not seem to play any role. A recent scenario attributes the rise of j_c to a pre-melting softening of the shear modulus C_{66} [12]. However, it is difficult to imagine any specific mechanism for the softening, especially that the rise of j_c is often *very* sharp (*e.g.*, see fig. 1 *b*)) and sometimes j_c even jumps discontinuously [17]. Here we propose that the rise of the critical current at the onset of the peak effect is a *consequence* of vortex-lattice melting in a weak random potential.

Thermal fluctuations cause melting of the vortex lattice [18], [19]. In ideal systems, the melting of a 2D algebraically ordered vortex lattice has been suggested to be either of the Kosterlitz-Thouless-Halperin-Nelson-Young type [18], or a weak first-order transition [20]. Much less is known for the melting of a 3D lattice in general. For a perfect 3D vortex lattice both analytic considerations [21] and numerical simulations [22] suggest a first-order transition. In particular, it has been shown that a finite density of free edge dislocations would result in a zero long-wavelength shear modulus [23]. In the presence of quenched random potentials, the ground state of the vortex array no longer possesses long-range translational order and the vortex array is pinned. However, when the quenched potential is weak the Larkin domains are very large. In the case of 3D, the translational correlation function could even be power laws over long distances (quasi–long-range order) [24]. Although there is no longer a melting transition of a long-range ordered lattice, one may still consider the disordering on a short length scale, or the melting of the Larkin domains [25]. Since, according to LO, it is the elastic length scale of a collective volume that determines the critical current, we show below that the melting of the Larkin domains may result in a rise of critical current in an experiment. We first consider the case of 2D where much is known for the melting of a pure lattice [26], [27]. We then speculate on the 3D case.

In a 2D elastic lattice, thermally excited dislocation pairs are bound for temperatures below the melting temperature T_m and, consequently, the shear modulus is finite. At T_m, the largest dislocation pairs start to dissociate and the long-wavelength shear modulus drops discontinuously to zero. Above T_m, the density of free dislocations rises from zero, and the mean distance between free dislocations is the Kosterlitz-Thouless correlation length $\zeta \sim \exp[c/(T-T_m)^{\bar{\nu}}]$ with $\bar{\nu} = 0.36963\cdots$ [27], which diverges as $T \to T_m^+$. The correlation length ζ also sets the length scale for the q(wave vector)-dependence of the shear modulus $\mu(q,T)$: roughly speaking, μ is zero for $q < 1/\zeta$ and finite for $q > 1/\zeta$. Now imagine that the vortex lattice is weakly pinned. For $T < T_m$, the Larkin length R_c sets the elastic length scale and the critical current density $j_c = n^{1/2}f/R_cB$. At $T = T_m$, the vortex lattice melts with the (infinitely) long wavelength shear modulus dropping to zero. However, the pinned lattice would not feel being melted at this point, as far as the j_c is concerned, since the lattice is elastically decoupled beyond the length scale of R_c by the quenched random potential. The critical current density is still determined by R_c. For $T > T_m$, another length scale ζ, the Kosterlitz-Thouless correlation length, enters the system. ζ decreases exponentially fast from the infinity as the temperature is increased and will soon become comparable to R_c. For $\zeta < R_c$, the Larkin domains decompose into smaller coherent domains and the relevant elastic length scale for the determination of j_c is now ζ:

$$j_c = \frac{n^{1/2}f}{\zeta B} \approx \frac{n^{1/2}f}{B} \exp\left[-\frac{c}{(T-T_m)^{\bar{\nu}}}\right], \ (\zeta < R_c). \tag{2}$$

Thus the onset of the peak effect occurs when the two length scales R_c *and* ζ *cross each other* (fig. 2). The exponential increase of j_c with T (eq. (2)) would continue until ζ is so small that

([2]) The peak effect in thin 2H-$NbSe_2$ crystals was previously interpreted as a dimensional crossover by Koorevaar *et al.* [16].

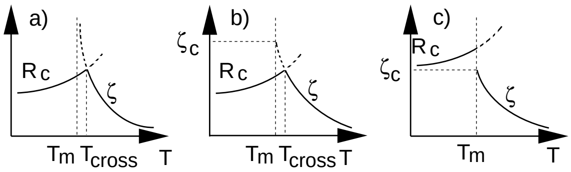

Fig. 2. – Schematic behavior of the two length scales R_c and ζ, as functions of temperature: *a*) continuous melting transition; *b*) first-order melting transition with $\zeta(T_m) > R_c(T_m)$; and *c*) first-order melting transition with $\zeta(T_m) < R_c(T_m)$. The temperature for the onset of the peak effect is T_{cross}, where ζ becomes smaller than R_c. In *c*), $T_{cross} = T_m$. The solid part of the lines determines the critical-current density. Note that the peak in these plots corresponds to the dip in j_c.

the thermally activated vortex motion starts to dominate, causing j_c to drop ([3]). In the very thin 2H-$NbSe_2$ crystal studied in ref. [4], the longitudinal Larkin pinning length L_c estimated from j_c is much longer than the thickness of the sample. Thus the pinning problem in this system is two-dimensional. The solid line in fig. 1 *b*) is a fit to the experimental data using eq. (2), assuming a simple linear relation $T - T_m \sim B - B_m$ and using the exact exponent $\bar{\nu} = 0.36963\cdots$. Given the simplicity of the model, the fit is remarkable. A numerical simulation in 2D [29] also indicated that the melting temperature is at the onset of the peak effect. Some other simulations suggested a very weak first-order melting transition in 2D [20]. In that case, one might expect to see a small jump in j_c (see below).

In the 3D case, melting of a pure lattice is much less understood. If the melting transition is mediated, or accompanied, by generation of the free edge dislocations, one would expect a similar mechanism for the peak effect as in 2D, with the elastic length ζ now being set by the mean distance between dislocation lines. Another length scale which may play the role of ζ is the viscous length [30]. To make a qualitative estimate for the behavior of j_c in the peak effect regime, we take the Landau-Ginzburg–like free energy often used in 3D dislocation systems [31]: $F(\rho) = -F_1\rho\ln C\rho + F_2\rho + F_3\rho^2$, where ρ is the areal density of dislocation lines, F_1 and F_3 are positive constants, C is a constant of the order a^2 with a being the lattice constant, $F_2 > 0$ at low temperatures and $F_2 < 0$ at high temperatures. The logarithmic term is due to the long-range elastic interaction of dislocation lines. It is easy to show that this free energy implies a first-order transition ([4]):

$$\rho = \begin{cases} 0, & T < T_m\,, \\ \rho_c + A(T - T_m), & T \geq T_m\,, \end{cases} \tag{3}$$

where $\rho_c = F_1/F_3$ and $A = [F_1'(\ln C\rho_c + 1) - F_2' - 2F_3'\rho_c]/F_3$. The mean distance between dislocation lines, $\zeta = \rho^{-1/2}$, is the length scale to be compared with the Larkin length R_c:

$$j_c = \frac{n^{1/2}f}{V_c^{1/2}B} = \begin{cases} \dfrac{n^{1/2}f}{L_c^{1/2}R_cB} \approx \dfrac{n^2f^4}{C_{66}^2C_{44}r_p^3B}, & \zeta > R_c\,, \\ \dfrac{n^{1/2}f}{L_c^{1/2}\zeta B} \approx \dfrac{n^{2/3}f^{4/3}\rho_c^{2/3}}{C_{44}^{1/3}r_p^{1/3}B}\Big[1 + \dfrac{A}{\rho_c}(T - T_m)\Big]^{2/3}, & \zeta < R_c\,, \end{cases} \tag{4}$$

([3]) Strictly speaking, there is no true critical current at finite temperature due to thermal activations [28]. One can, however, combine Anderson's picture of thermal activation with LO's collective pinning theory to obtain a *measured* j_c as $j_cB = (nf^2/V_c)^{1/2} - \alpha k_BT/r_pV_c$, where α is determined by experimental sensitivity. For weak pinning where V_c is large, thermal activation (the second term) is insignificant until the lattice melts.

([4]) We are grateful to M. Rabin for pointing out an error in our original calculation.

where in the region of $\zeta < R_c$ eq. (1) is minimized, with R_c replaced by ζ, to determine L_c. Since ζ has a discontinuous jump at T_m (if the transition is first order), it is possible that j_c will have a jump at the onset of the peak effect which may occur in samples with "very weak" pinning (fig. 2*c*)). In fact, a jump in j_c was observed experimentally in a 3D sample [17].

To conclude, we have proposed a mechanism for the peak effect based on the spontaneous generation of free dislocations closely related to the melting of the vortex lattice in the presence of weak pinning. The onset of the peak effect is the crossover of the two elastically relevant length scales. Note that our argument does not depend on the detailed nature of the melting transition, *e.g.*, first *vs.* second order. We believe that this picture captures the basic physics of the peak effect, at least for weak enough pinning. On the other hand, if the pinning is so strong that the Larkin length is of the order of the lattice constant, the sample should not, according to our scenario, show the peak effect.

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