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An Analytical Model for Performance Evaluation of Operating Room Schedules in Orthopedic Surgery

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Abstract— **In orthopedic surgery department, multiple surgeries are carried out in the same operating room every day. Each surgery may have a random duration, differs from the scheduled time, which results in room idle time and patient waiting time. One of the major factors affecting the idle and waiting times is the schedule of surgeries in the operating room. To better sequence of surgeries to reduce idle time and waiting time, an effective performance evaluation method is needed. Although discrete event simulation can be used to evaluate the performance of surgery schedules, it suffers from long simulation time, which makes difficult in scheduling optimization. In this paper, an analytical model to evaluate the performance of operating room schedules in orthopedic surgery department is introduced. Using such a model, the expected idle time and waiting time for a given surgery schedule can be calculated accurately. Such a work provides substantial easiness for optimization of operating room schedules and investigation of the impact of different schedules.**

Keywords: Operating room, orthopedic surgery, idle time, waiting time, scheduling, optimization.

I. INTRODUCTION

The operating room (OR), or operating theater, is the hospitals largest cost center, but it also generates about 42 percent of a hospital's revenue [1]. The hospital's income and performance are significantly affected by the OR. However, managing the OR is difficult due to the complexity in surgeries, resources and high cost. It also requires a wide range of clinical and organizational skills to address the needs of patients, families and support systems. For each surgery, multiple tasks need to be accomplished to ensure all preparations, anesthesia, operating and post anesthesia care are satisfied. Significant amounts of attention has been paid from the hospital's governing body and from researchers to organize surgical care with least cost, within which OR scheduling is one of the central issues.

Substantial efforts have been devoted to operating room scheduling to address the different phases and aspects (see, for instance, reviews [2]-[5]). The focuses are mainly on improving the theaters efficiency, turnover rate, patient outcome, and surgical department capacity. For example, paper [6] uses linear programming technique to optimize the allocation of OR time among a group of surgeons, which can maximize revenue or minimize costs substantially. Paper [7] presents a stochastic optimization model and practical heuristics to compute OR schedules that hedge against the uncertainties in surgery durations. Paper [8] uses discrete event simulation to study reserved surgical capacity for emergency department. Using the job shop scheduling method, paper [9] proposes a scheduling approach for elective and add-on surgeries by formulating it as a mixed integer linear programming problem. Aiming at open scheduling strategy, paper [10] constructs a mathematical model assign surgical cases to operating rooms and proposes a column-generation-based heuristic procedure to find a solution with the best performance. By considering patient priority, paper [11] presents a stochastic dynamic programming model to schedule elective surgery with a limited capacity.

In spite of the efforts, the surgery scheduling problem is still of interests. Efficient algorithms to achieve optimal utilization of operating rooms and reduced waiting time are needed. However, this depends on accurate evaluation of schedule performance, such as room idle time, patient waiting time, etc. Typically, either deterministic models or discrete event simulations are used in performance evaluation of OR schedules. This leads to either ignore the variability and resulting idle and waiting times, or substantial computation intensity which limits its application. An effective method to evaluate the performance of OR schedules quickly and accurately is necessary and important. Unfortunately, to our best knowledge, an analytical model to achieve such a function is still not available. The main contribution of this paper is in developing such a method. Specifically, an aggregation method is presented to approximate the room idle time and patient waiting time for a given surgery sequence, defined by mean and coefficient of variations. By comparing with simulation models using the data collected on the hospital floor at University of Wisconsin Hospital and Clinics (UW Health), the effectiveness of the proposed method is justified.

The remainder of this paper is structured as follows: Section II introduces the system and formulates the problem. The performance evaluation method is presented in Section III and validated in Section IV. Finally, conclusions and future work are provided in Section V.

II. SYSTEM DESCRIPTION AND PROBLEM FORMULATION

Due to the complexity of surgeries, when multiple surgeries are scheduled in one room, it is not uncommon to

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observe that one surgery finishes earlier than next surgery's appointment time. The theater and other resources remain idle and unutilized until the appointment time when next surgery starts. It is also common to see one surgery finishing later than next surgery's appointment time. Therefore, patients and resources for later appointments have to wait.

To reduce room idle time and patient waiting time, scheduling the sequence of surgeries plays a key role. To study such an issue, we focus on daily surgery scheduling in one operating room. Each surgery is modeled with a random processing duration given by a probability density function describing the surgery type. Then the goal is to develop a method to evaluate room idle and patient waiting times for a given surgery sequence.

A. Assumptions

The following assumptions address the orthopedic surgery schedules:

1) There are N surgeries, S_1, S_2, ..., S_N, to be scheduled in one orthopedic operating room per day.
2) All patients arrive in-time, i.e., the patients either arrive on time or earlier. No patients will arrive later than the scheduled surgery time.
3) All pre-surgery operations will be finished by the scheduled surgery starting time. In other words, a surgery will not be delayed due to incomplete preparation.
4) The turnover time between surgeries is included in the surgery duration.
5) The surgeries are scheduled based on the mean surgery time (including the turnover time).
6) The duration of each surgery is described by a Gamma distribution with parameters α_i and β_i, $i = 1, \ldots, N$.
7) If a surgery finishes earlier than the scheduled time, the next surgery will not start until the appointment time. The gap between two surgeries is referred to as *idle time*.
8) If a surgery finished later than the scheduled time, the next surgery will start immediately. The overtime of the previous surgery contributes to the *waiting time*.
9) The first surgery always starts on time. No scheduled surgery will be cancelled or postponed to other dates.

Remark 1: Different type of surgeries may be fitted by different probabilistic distributions. To start with, Gamma distribution is assumed to model all types of surgeries, as it has two parameters, which provides the freedom to select mean and variance. In Section IV, we justify that the system performance is mainly dependent on the mean and coefficient of variation, in other words, it does not depend on distribution type, and using Gamma distribution does lead to accurate estimation of waiting time and idle time.

B. Problem Formulation

To evaluate the performance of OR scheduling, patient waiting time and room idle time are important measurements. Introduce T_{w_i} as the patient waiting time for the i-th surgery, $i = 2, \ldots, N$, i.e., from the scheduled starting time of surgery i to its actual starting time, in case of late finishing of surgery $i-1$. Let T_{e_i} denote the room idle time of surgery i, $i = 1, \ldots, N-1$, i.e., from the time surgery S_i finishes to the scheduled starting time of surgery S_{i+1}, during which the room is empty of patient. Then the total waiting time and idle time are the summation of its corresponding time of each surgery.

$$T_w = \sum_{i=2}^{N} T_{w_i}, \quad (1)$$

$$T_e = \sum_{i=1}^{N-1} T_{e_i}. \quad (2)$$

Then the problem is formulated as follows: *Under assumptions 1)-9), develop a method to evaluate the room idle time T_e and patient waiting time T_w.*

Remark 2: In addition to idle time and waiting time, the completion time of all surgeries and its variance, are also of interests. Let C_i represent the average time to complete surgeries 1 to i, $i = 2, \ldots, N$, and V_i as its variance. Evaluating C_i and V_i will also be pursued.

The solutions to the formulated problem are introduced next.

III. PERFORMANCE EVALUATION METHOD

The goal of this paper is to develop a method to calculate the surgery completion time, variance, and the patient waiting time and room idle time when multiple surgeries are scheduled in one operating room. Under assumption 6), the duration of surgery S_i, $i = 1, \ldots, N$, has parameters α_i and β_i following Gamma distribution,

$$g(x; \alpha_i, \beta_i) = \frac{\beta_i^{\alpha_i}}{\Gamma(\alpha_i)} x^{\alpha_i - 1} e^{-\beta_i x},$$

where $\Gamma(s)$ is the gamma function,

$$\Gamma(s) = \int_0^\infty t^{s-1} e^{-t} dt.$$

In addition, the mean duration, τ_i, equals to $\frac{\alpha_i}{\beta_i}$, and the variance, V_i, equals to $\frac{\alpha_i}{\beta_i^2}$.

To evaluate the system performance, we start with a two-surgery scenario, and then extend to N surgeries.

A. Two Surgeries

First we consider the case that surgery S_1 finishes before the scheduled time (i.e., mean time τ_1). Such a probability, p_{e_1}, can be calculated as

$$p_{e_1} = \int_0^{\tau_1} g(x) dx = \frac{\gamma(\alpha_1, \alpha_1)}{\Gamma(\alpha_1)}, \quad (3)$$

where $\gamma(s, x)$ is the lower incomplete gamma function,

$$\gamma(s, x) = \int_0^x t^{s-1} e^{-t} dt.$$

Under this condition, since the second surgery S_2 will not start until the scheduled starting time, it is equivalent to view that S_1 still "completes at time τ_1." Therefore, in this early finishing case, the mean duration of finishing two surgeries (S_1 and S_2), C_{e_2}, will be

$$C_{e_2} = \tau_1 + \tau_2 = \frac{\alpha_1}{\beta_1} + \frac{\alpha_2}{\beta_2}.$$

To evaluate the variance of duration of S_1 and S_2, note that S_1 is viewed as a constant without variance, and the two surgeries are consecutive. Thus, the total variance, V_{e_2}, equals to the variance of the second surgery, i.e.,

$$V_{e_2} = \frac{\alpha_2}{\beta_2^2}.$$

Using $Mathematica$, the average room idle time can be derived as follows:

$$\begin{aligned} T_{e_1} &= \int_0^{\tau_1} (\tau_1 - x) f(x) dx \\ &= \frac{-\alpha_1 \zeta(\alpha_1, \beta_1\tau_1) + \zeta(1+\alpha_1, \beta_1\tau_1)}{\beta_1 \Gamma(\alpha_1)}, \end{aligned} \quad (4)$$

where $\zeta(s, x)$ is the upper incomplete gamma function,

$$\zeta(s, x) = \int_x^{\infty} t^{s-1} e^{-t} dt.$$

Remark 3: Note that the above integral and much of the subsequent derivation are obtained using $Mathematica$.

Next we consider the scenario that surgery S_1 goes over the scheduled finishing time τ_1. The probability of such an event is

$$p_{w_1} = \int_{\tau_1}^{\infty} g(x) dx = \frac{\zeta(\alpha_1, \alpha_1)}{\Gamma(\alpha_1)}. \quad (5)$$

Then the mean time of surgery S_1 will be equal to

$$\int_{\tau_1}^{\infty} x (g(x)/p_{w_1}) dx = \frac{\zeta(1+\alpha_1, \alpha_1)}{\beta_1 \zeta(\alpha_1, \alpha_1)}.$$

As S_1 is over time, surgery S_2 will start immediately after S_1. Then the mean completion time of both surgeries S_1 and S_2 in late finishing case, C_{w_2}, will be

$$C_{w_2} = \frac{\zeta(1+\alpha_1, \alpha_1)}{\beta_1 \zeta(\alpha_1, \alpha_1)} + \frac{\alpha_2}{\beta_2}.$$

The variance of surgery S_1 will be equal to

$$\begin{aligned} &\left(\int_{\tau_1}^{\infty} x^2 (g(x)/p_{w_1}) dx\right) - \left(\frac{\zeta(1+\alpha_1, \alpha_1)}{\beta_1 \zeta(\alpha_1, \alpha_1)}\right)^2 \\ &= \frac{-\zeta^2(1+\alpha_1, \alpha_1) + \zeta(\alpha_1, \alpha_1)\zeta(2+\alpha_1, \alpha_1)}{\beta_1^2 \zeta^2(\alpha_1, \alpha_1)}. \end{aligned}$$

The variance of both surgeries in this scenario, V_{w_2}, will be

$$V_{w_2} = \frac{-\zeta^2(1+\alpha_1, \alpha_1) + \zeta(\alpha_1, \alpha_1)\zeta(2+\alpha_1, \alpha_1)}{\beta_1^2 \zeta^2(\alpha_1, \alpha_1)} + \frac{\alpha_2}{\beta_2^2}.$$

The average patient waiting time can also be calculated:

$$\begin{aligned} T_{w_2} &= \int_{\tau_1}^{\infty} (x - \tau_1) f(x) dx \\ &= \frac{-\beta_1\tau_1 \zeta(\alpha_1, \beta_1\tau_1) + \zeta(1+\alpha_1, \beta_1\tau_1)}{\beta_1 \Gamma(\alpha_1)}. \end{aligned} \quad (6)$$

By considering both scenarios, we obtain the mean surgery completion time C_2 and variance V_2 for both surgeries.

$$\begin{aligned} C_2 &= C_{e_2} p_{e_1} + C_{w_2} p_{w_1} \\ &= \frac{\zeta(1+\alpha_1, \beta_1\tau_1) - \beta_1\tau_1 \zeta(\alpha_1, \beta_1\tau_1)}{\beta_1 \Gamma(\alpha_1)} \\ &\quad + \frac{\alpha_1}{\beta_1} + \frac{\alpha_2}{\beta_2}, \end{aligned} \quad (7)$$

$$\begin{aligned} V_2 &= V_{e_2} p_{e_1} + V_{w_2} p_{w_1} \\ &= \frac{-\zeta^2(1+\alpha_1, \beta_1\tau_1) + \zeta(\alpha_1, \beta_1\tau_1)\zeta(2+\alpha_1, \beta_1\tau_1)}{\beta_1^2 \Gamma(\alpha_1) \zeta(\alpha_1, \beta_1\tau_1)} \\ &\quad + \frac{\alpha_2}{\beta_2^2}. \end{aligned} \quad (8)$$

B. Three Surgeries

When there are more than two surgeries, direct integral to derive the idle time and waiting time is difficult, since the number of different scenarios will increase substantially. Therefore, an approximation method is pursued. To do this, we aggregate the first two surgeries into one, and assume that this aggregated surgery still follows a Gamma distribution. In other words, S_{a_2} represents the aggregated surgery of both surgeries S_1 and S_2. The mean and variance of surgery S_{a_2} are defined by C_2 and V_2 obtained in the Subsection III-A. Then parameters α_{a_2} and β_{a_2} of surgery S_{a_2} can be obtained (see formulas on next page).

Using S_{a_2} and S_3, the completion time of three surgeries C_3, and the variance V_3, can also be evaluated.

$$\begin{aligned} C_3 &= \frac{\zeta(1+\alpha_{a_2}, \beta_{a_2}\tau_{s_2}) - \beta_{a_2}\tau_{s_2} \zeta(\alpha_{a_2}, \beta_{a_2}\tau_{s_2})}{\beta_{a_2} \Gamma(\alpha_{a_2})} \\ &\quad + \tau_{s_2} + \frac{\alpha_3}{\beta_3}, \end{aligned} \quad (11)$$

$$\begin{aligned} V_3 &= \frac{\alpha_3}{\beta_3^2} + \frac{\zeta(2+\alpha_{a_2}, \beta_{a_2}\tau_{s_2})}{\beta_{a_2}^2 \Gamma(\alpha_{a_2})} \\ &\quad - \frac{\zeta^2(1+\alpha_{a_2}, \beta_{a_2}\tau_{s_2})}{\beta_{a_2}^2 \Gamma(\alpha_{a_2}) \zeta(\alpha_{a_2}, \beta_{a_2}\tau_{s_2})}. \end{aligned} \quad (12)$$

where

$$\tau_{s_2} = \tau_1 + \tau_2 = \frac{\alpha_1}{\beta_1} + \frac{\alpha_2}{\beta_2}. \quad (13)$$

However, C_3 cannot be used to evaluate idle time and waiting time directly, since they depend on many combinations of finishing time of S_1 and S_2. For instance, there will be both idle and waiting times if S_1 finishes before the scheduled starting time of S_2, but S_2 ends after the scheduled starting time. Thus, we need to compare the completion time with the scheduled finishing time.

As we know, the scheduled finishing time of surgeries S_1 and S_2 will be $\tau_1 + \tau_2$, denoted as τ_{s_2}, i.e., the scheduled finishing time of the aggregated surgery. Then, using formulas (6) and (4), by replacing τ_1 with τ_{s_2}, α_1 with α_{a_2}, and β_1 with β_{a_2}, the room idle time of surgery S_2 and patient waiting time for surgery S_3 can be calculated. Thus, using α_{a_2} and β_{a_2} obtained from (9) and (10), respectively, the formulas for T_{e_2} and T_{w_3} are provided (on next page).

Then the total waiting time and idle time can be obtained by summing up the waiting time of surgeries S_2 and S_3, and

$$\alpha_{a_2} = \frac{C_2^2}{V_2} = \frac{\zeta(\alpha_1,\beta_1\tau_1)[\beta_1(\alpha_2+\beta_2\tau_1)\Gamma(\alpha_1)+\beta_2(-\beta_1\tau_1\zeta(\alpha_1,\beta_1\tau_1)+\zeta(1+\alpha_1,\beta_1\tau_1))]^2}{\Gamma(\alpha_1)[\alpha_2\beta_1^2\Gamma(\alpha_1)\zeta(\alpha_1,\beta_1\tau_1)+\beta_2^2(-\zeta^2(1+\alpha_1,\beta_1\tau_1)+\zeta(\alpha_1,\beta_1\tau_1)\zeta(2+\alpha_1,\beta_1\tau_1))]}, \quad (9)$$

$$\beta_{a_2} = \frac{C_2}{V_2} = \frac{\beta_1\beta_2\zeta(\alpha_1,\beta_1\tau_1)[\beta_1(\alpha_2+\beta_2\tau_1)\Gamma(\alpha_1)+\beta_2(-\beta_1\tau_1\zeta(\alpha_1,\beta_1\tau_1)+\zeta(1+\alpha_1,\beta_1\tau_1))]}{\alpha_2\beta_1^2\Gamma(\alpha_1)\zeta(\alpha_1,\beta_1\tau_1)+\beta_2^2[-\zeta^2(1+\alpha_1,\beta_1\tau_1)+\zeta(\alpha_1,\beta_1\tau_1)\zeta(2+\alpha_1,\beta_1\tau_1)]}. \quad (10)$$

$$\begin{aligned} T_{e_2} &= \int_0^{\tau_{s_2}}(\tau_{s_2}-x)f(x)dx = \int_0^{\tau_{s_2}}(\tau_{s_2}-x)\frac{\beta_{a_2}^{\alpha_{a_2}}}{\Gamma(\alpha_{a_2})}x^{\alpha_{a_2}-1}e^{-\beta_{a_2}x}dx \\ &= \frac{(-\alpha_{a_2}+\beta_{a_2}\tau_{s_2})\Gamma(\alpha_{a_2})-\beta_{a_2}\tau_{s_2}\zeta(\alpha_{a_2},\beta_{a_2}\tau_{s_2})+\zeta(1+\alpha_{a_2},\beta_{a_2}\tau_{s_2})}{\beta_{a_2}\Gamma(\alpha_{a_2})}, \quad (14) \\ T_{w_3} &= \int_{\tau_{s_2}}^{\infty}(x-\tau_{s_2})f(x)dx = \int_{\tau_{s_2}}^{\infty}(x-\tau_{s_2})\frac{\beta_{a_2}^{\alpha_{a_2}}}{\Gamma(\alpha_{a_2})}x^{\alpha_{a_2}-1}e^{-\beta_{a_2}x}dx \\ &= \frac{-\beta_{a_2}\tau_{s_2}\zeta(\alpha_{a_2},\beta_{a_2}\tau_{s_2})+\zeta(1+\alpha_{a_2},\beta_{a_2}\tau_{s_2})}{\beta_{a_2}\Gamma(\alpha_{a_2})}. \quad (15) \end{aligned}$$

idle time of surgeries S_1 and S_2, respectively. Thus,

$$\begin{aligned} T_w &= T_{w_2} + T_{w_3}, \\ T_e &= T_{e_1} + T_{e_2}. \end{aligned}$$

The final expressions of T_w and T_e are obtained (see next page).

In order to provide formulas to evaluate the performance of four surgeries, again by aggregating surgery S_{a_2} and surgery S_3 into one, and assuming Gamma distribution, we obtain the parameters α_{a_3} and β_{a_3} for the aggregated surgery S_{a_3} (see next page).

C. N Surgeries

Using the similar approach, we can extend the study to N surgeries. Specifically, the following iteration procedure is proposed:

- Aggregate surgeries S_1 and S_2 into S_{a_2}. Calculate its mean surgery time C_2 and variance V_2, idle time T_{e_1} and waiting time T_{w_2}, and obtain parameters α_{a_2} and β_{a_2}.
- Aggregate surgeries S_{a_2} and S_3 into S_{a_3}. Using parameters α_{a_2} and β_{a_2} to calculate C_3, V_3, T_{e_2}, and T_{w_3}. Obtain new parameters α_{a_3} and β_{a_3}.
- Repeat this process until surgery N, i.e., aggregate surgery S_{a_i} (using parameters α_{a_i} and β_{a_i} from previous step) and surgery S_{i+1} into a new aggregated surgery $S_{a_{i+1}}$, with parameters $\alpha_{a_{i+1}}$ and $\beta_{a_{i+1}}$, till $i = N-1$. Calculate C_{i+1}, V_{i+1}, T_{e_i} and $T_{w_{i+1}}$.
- Calculate surgery completion time C_N and variance V_N, and the overall idle time T_e and waiting time T_w.

An illustration of such a procedure for a four-surgery schedule is shown in Figure 1. A formal expression of such a procedure is shown in (20)-(26) (on next page).

Finally, from (1) and (2), the total waiting time and idle time can be calculated.

IV. MODEL VALIDATION

To validate the model, extensive simulation experiments have been carried out using a commercial software *SIMUL8*

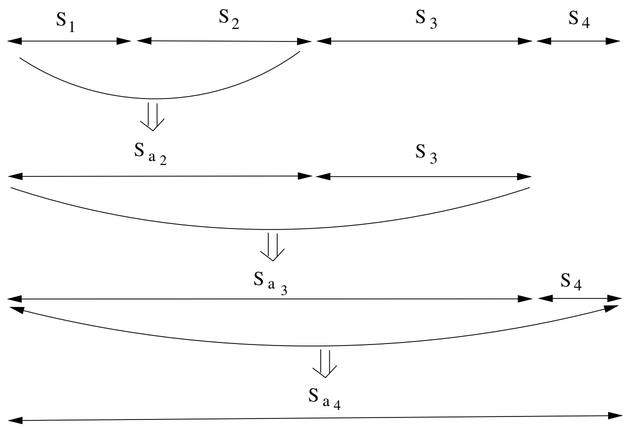

Fig. 1. Illustration of aggregation procedure

[12]. In all simulations, each experiment simulates 60 days with 1000 replications. The system parameters are randomly selected from:

$$\begin{aligned} N &\in \{2,3,4,5\}, \\ \tau_i &\in \{40,80,120\}, \quad i=1,\ldots,5, \\ cv_i &\in \{0.2,0.4,0.6,0.8\}, \quad i=1,\ldots,5, \end{aligned}$$

where cv_i denotes the coefficient of variation (CV) of surgery time for S_i. In all experiments, the confidence intervals are within 1.5% of the performance measure.

A. Accuracy of Aggregation Procedure

First, we validate the accuracy of aggregation procedure. Assumption 6) assumes that all events are characterized by Gamma distribution. The analytical formulas assume that the aggregated event is also described by Gamma. To validate the effectiveness of such an approximation, the results obtained in Section III will be compared with simulation results where all surgeries follow Gamma distributions, to verify that the aggregation approximation has sufficient accuracy.

Let T_w^{sim}, T_e^{sim}, and T_w^{mod} and T_e^{mod} denote the average waiting time and idle time obtained by simulation and

$$T_w = \frac{-\beta_1\tau_1\zeta(\alpha_1,\beta_1\tau_1)+\zeta(1+\alpha_1,\beta_1\tau_1)}{\beta_1\Gamma(\alpha_1)} + \frac{-\beta_{a_2}\tau_{s_2}\zeta(\alpha_{a_2},\beta_{a_2}\tau_{s_2})+\zeta(1+\alpha_{a_2},\beta_{a_2}\tau_{s_2})}{\beta_{a_2}\Gamma(\alpha_{a_2})}, \tag{16}$$

$$T_e = \frac{-\alpha_1\zeta(\alpha_1,\beta_1\tau_1)+\zeta(1+\alpha_1,\beta_1\tau_1)}{\beta_1\Gamma(\alpha_1)} + \frac{(-\alpha_{a_2}+\beta_{a_2}\tau_{s_2})\Gamma(\alpha_{a_2})-\beta_{a_2}\tau_{s_2}\zeta(\alpha_{a_2},\beta_{a_2}\tau_{s_2})+\zeta(1+\alpha_{a_2},\beta_{a_2}\tau_{s_2})}{\beta_{a_2}\Gamma(\alpha_{a_2})}. \tag{17}$$

$$\alpha_{a_3} = \frac{\zeta(\alpha_{a_2},\beta_{a_2}\tau_{s_2})[\beta_{a_2}(\alpha_3+\beta_3\tau_{s_2})\Gamma(\alpha_{a_2})+\beta_3(-\beta_{a_2}\tau_{s_2}\zeta(\alpha_{a_2},\beta_{a_2}\tau_{s_2})+\zeta(1+\alpha_{a_2},\beta_{a_2}\tau_{s_2}))]^2}{\Gamma(\alpha_{a_2})[\alpha_3\beta_{a_2}^2\Gamma(\alpha_{a_2})\zeta(\alpha_{a_2},\beta_{a_2}\tau_{s_2})+\beta_3^2(-\zeta^2(1+\alpha_{a_2},\beta_{a_2}\tau_{s_2})+\zeta(\alpha_{a_2},\beta_{a_2}\tau_{s_2})\zeta(2+\alpha_{a_2},\beta_{a_2}\tau_{s_2}))]}, \tag{18}$$

$$\beta_{a_3} = \frac{\beta_{a_2}\beta_3\zeta(\alpha_{a_2},\beta_{a_2}\tau_{s_2})[\beta_{a_2}(\alpha_3+\beta_3\tau_{s_2})\Gamma(\alpha_{a_2})+\beta_3(-\beta_{a_2}\tau_{s_2}\zeta(\alpha_{a_2},\beta_{a_2}\tau_{s_2})+\zeta(1+\alpha_{a_2},\beta_{a_2}\tau_{s_2}))]}{\alpha_3\beta_{a_2}^2\Gamma(\alpha_{a_2})\zeta(\alpha_{a_2},\beta_{a_2}\tau_{s_2})+\beta_3^2[-\zeta^2(1+\alpha_{a_2},\beta_{a_2}\tau_{s_2})+\zeta(\alpha_{a_2},\beta_{a_2}\tau_{s_2})\zeta(2+\alpha_{a_2},\beta_{a_2}\tau_{s_2})]}. \tag{19}$$

Procedure 1:

$$T_{e_i} = \frac{(-\alpha_{a_i}+\beta_{a_i}\tau_{s_i})\Gamma(\alpha_{a_i})-\beta_{a_i}\tau_{s_i}\zeta(\alpha_{a_i},\beta_{a_i}\tau_{s_i})+\zeta(1+\alpha_{a_i},\beta_{a_i}\tau_{s_i})}{\beta_{a_i}\Gamma(\alpha_{a_i})}, \tag{20}$$

$$T_{w_{i+1}} = \frac{-\beta_{a_i}\tau_{s_i}\zeta(\alpha_{a_i},\beta_{a_i}\tau_{s_i})+\zeta(1+\alpha_{a_i},\beta_{a_i}\tau_{s_i})}{\beta_{a_i}\Gamma(\alpha_{a_i})}, \tag{21}$$

$$C_{i+1} = \tau_{s_i} + \frac{\alpha_{i+1}}{\beta_{i+1}} + \frac{\zeta(1+\alpha_{a_i},\beta_{a_i}\tau_{s_i})-\beta_{a_i}\tau_{s_i}\zeta(\alpha_{a_i},\beta_{a_i}\tau_{s_i})}{\beta_{a_i}\Gamma(\alpha_{a_i})}, \tag{22}$$

$$V_{i+1} = \frac{-\zeta^2(1+\alpha_{a_i},\beta_{a_i}\tau_{s_i})+\zeta(\alpha_{a_i},\beta_{a_i}\tau_{s_i})\zeta(2+\alpha_{a_i},\beta_{a_i}\tau_{s_i})}{\beta_{a_i}^2\Gamma(\alpha_{a_i})\zeta(\alpha_{a_i},\beta_{a_i}\tau_{s_i})} + \frac{\alpha_{i+1}}{\beta_{i+1}^2}, \tag{23}$$

$$i = 1,2,\ldots,N-1,$$

where

$$\tau_{s_i} = \sum_{j=1}^{i}\tau_i = \sum_{j=1}^{i}\frac{\alpha_j}{\beta_j}, \tag{24}$$

$$\alpha_{a_{i+1}} = \frac{\zeta(\alpha_{a_i},\beta_{a_i}\tau_{s_i})[\beta_{a_i}(\alpha_{i+1}+\beta_{i+1}\tau_{s_i})\Gamma(\alpha_{a_i})+\beta_{i+1}(-\beta_{a_i}\tau_{s_i}\zeta(\alpha_{a_i},\beta_{a_i}\tau_{s_i})+\zeta(1+\alpha_{a_i},\beta_{a_i}\tau_{s_i}))]^2}{\Gamma(\alpha_{a_i})[\alpha_{i+1}\beta_{a_i}^2\Gamma(\alpha_{a_i})\zeta(\alpha_{a_i},\beta_{a_i}\tau_{s_i})+\beta_{i+1}^2(-\zeta^2(1+\alpha_{a_i},\beta_{a_i}\tau_{s_i})+\zeta(\alpha_{a_i},\beta_{a_i}\tau_{s_i})\zeta(2+\alpha_{a_i},\beta_{a_i}\tau_{s_i}))]}, \tag{25}$$

$$\beta_{a_{i+1}} = \frac{\beta_{a_i}\beta_{i+1}\zeta(\alpha_{a_i},\beta_{a_i}\tau_{s_i})[\beta_{a_i}(\alpha_{i+1}+\beta_{i+1}\tau_{s_i})\Gamma(\alpha_{a_i})+\beta_{i+1}(-\beta_{a_i}\tau_{s_i}\zeta(\alpha_{a_i},\beta_{i+1}\tau_{s_i})+\zeta(1+\alpha_{a_i},\beta_{a_i}\tau_{s_i}))]}{\alpha_{i+1}\beta_{a_i}^2\Gamma(\alpha_{a_i})\zeta(\alpha_{a_i},\beta_{a_i}\tau_{s_i})+\beta_{i+1}^2[-\zeta^2(1+\alpha_{a_i},\beta_{a_i}\tau_{s_i})+\zeta(\alpha_{a_i},\beta_{a_i}\tau_{s_i})\zeta(2+\alpha_{a_i},\beta_{a_i}\tau_{s_i})]}. \tag{26}$$

analytical model, respectively. Introduce

$$\delta_w = \frac{|T_w^{sim}-T_w^{mod}|}{T_w^{sim}}\cdot 100\%,$$

$$\delta_e = \frac{|T_e^{sim}-T_e^{mod}|}{T_e^{sim}}\cdot 100\%.$$

Based on about five dozen experiments, the differences of waiting time estimation, δ_w, are all within 5%, while the average accuracy is 1.32%. The differences of idle time estimation, δ_e, are less than 7%, and the average accuracy is 1.94%. An illustration example is shown in Figure 2, where four surgeries are scheduled. The mean duration of each surgery is selected from 40, 80, and 120 minutes. The first surgery has $CV = 0.6$, while the second surgery $CV = 0.4$. Figure 2 illustrates δ_w and δ_e as a function of the CV of the third surgery. As one can see, the differences are consistently around 1-2% for waiting time, and varies around 2-4% for idle time.

The above results suggest that the aggregation approach introduced here has sufficient accuracy to estimate the idle time and waiting time of surgery schedules.

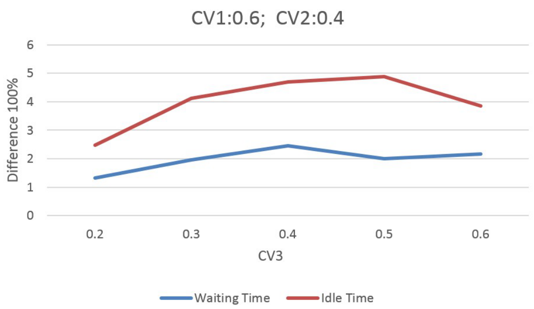

Fig. 2. A four-surgery example

B. Accuracy of Analytical Model

Next, we validate the effectiveness of analytical model by using the data collected on the orthopedic surgery operating rooms at University of Wisconsin Hospital and Clinics (UW Health). The data is collected from the events in 2012. Ten surgical types from more than 5200 events are included in the study and summarized in Table I.

TABLE I

10 SURGICAL TYPES

Type	Number	Mean	STD	CV
Hand upper extremity	738	70.65	50.69	0.7175
Spine	531	171.2	92.51	0.5383
Joint	944	149.44	47.97	0.3210
Sports medicine	1202	107.76	59.47	0.5519
Foot & ankle	73	165.99	60.67	0.3655
General/plastics	257	138.14	107.23	0.7762
General/bariatric	946	123.35	57.03	0.4623
General/hernia	122	155.65	118.05	0.7584
Urology	218	106.85	74.14	0.6939
Spine neuro	238	186.45	122.24	0.6556

As one can see, the surgery time varies substantially with surgical types, and the variation of surgical time in each type also varies with CV between 0.32 to 0.78. For each surgery type, the collected data will be fitted into a distribution using the Stat Fit package in *SIMUL8*. The resulting distribution types include: Gamma, Weibull, Log-normal, Triangular, Beta, Pearson V, and Pearson VI, etc. In each experiment, we randomly select number of surgeries, and randomly select the surgical type from Table I. The fitted distribution for the selected surgery type will be used in simulation model. The mean and standard deviation are provided to the analytical model. Then δ_w and δ_e are evaluated using the same approach as in Subsection IV-A.

Based on about five dozens experiments we obtain that the differences of waiting time estimation, δ_w, are all within 6%, while the average accuracy is 3.40%. The differences of idle time estimation, δ_e, are within 9%, and the average accuracy is 5.34%. Such results indicate that the developed model has acceptable accuracy in estimating patient waiting time and room idle time for a given surgery schedule. In addition, since only Gama distribution is assumed in the analytical model, while more than half dozen different distribution types have been used in fitting the simulation model, we hypothesize that the system performance is practically independent of distribution type, but mainly depends on the mean and CV. In other words, the analytical model is suitable for a general distribution of surgery times. Similar properties have been observed in healthcare clinics ([13], [14]), as well as in manufacturing systems ([15]).

V. CONCLUSIONS

This paper introduce an analytical model to evaluate the performance of operating room schedules in orthopedic surgery. The room idle time and patient waiting time can be calculated for two-surgery schedule. Using an aggregation approach, every two surgeries can be represented by an aggregated surgery. Through an iterative procedure, multiple surgery schedules can be evaluated and the idle time and waiting time can be estimated. Using the data collected on the hospital floor in UW Health, numerical experiments have shown that such a method results in acceptable accuracy and can be effectively used for performance evaluation of surgery schedules. Such a model provides the healthcare professionals a quantitative tool for optimization and improvement in operating room scheduling.

In future work, we expect to extend the work in the following directions:

- extend the study to other types of surgeries,
- develop methods for surgery scheduling optimization,
- investigate the fundamental properties and principles in surgery scheduling, and finally,
- apply the model in daily scheduling activities on the hospital floor. A preliminary version of Java software suite to implement the algorithms has been developed and will be deployed at UW Health.

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