Improving Discharge Process at the University of Wisconsin Hospital: A System-Theoretic Method

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***Abstract*— This paper introduces a system-theoretic approach to improve inpatient discharge process at the University of Wisconsin (UW) Hospital. The complex hospital discharge process is modeled by a stochastic process with parallel subprocesses, splits, merges, and reworks. Then, a stochastic analysis method is introduced to evaluate the performance of discharge. Specifically, the waiting and service times are characterized by gamma distributions, and an efficient algorithm is presented to aggregate the multiple interacting subprocesses and calculate the mean, variability, and discharge-time performance, i.e., the probability to discharge a patient within a desired or given time interval. High accuracy in performance evaluation is obtained by using such a method. To improve the discharge process at UW Hospital, bottleneck and what-if analyses are carried out and improvement recommendations are discussed.**

***Note to Practitioners*—Hospital discharge is a complex process with substantial variabilities and challenges. Delays in discharge are common and become a nationwide problem. Although significant efforts have been devoted to improving the discharge process, most of the studies are qualitative or case specific. Thus, there is a need to develop a mathematical model to systematically characterize the discharge process, analyze the performance, and investigate improvement options, which can provide a fresh look from the system perspective. In this paper, we introduce a system-theoretic approach to evaluate and improve the inpatient discharge process at the University of Wisconsin (UW) Hospital. The discharge process is represented by a complex network consisting of multiple stochastic subprocesses, as well as splits, merges, and reworks. In addition to evaluating mean and variance of the discharge time, we introduce the notion of discharge-time performance (DTP), which is the probability to discharge a patient within a desired time interval, and an approximation method to calculate DTP. Moreover, we carry out a bottleneck analysis to identify the most critical step impeding the discharge process in the strongest manner. We also present recommendations to improve the discharge process at UW Hospital.**

***Index Terms*— Aggregation, bottleneck, discharge process, discharge-time performance (DTP), improvement, stochastic model.**

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I. Introduction

According to the National Hospital Discharge Survey, there are about 35 million discharges in the United States annually [1]. Due to the extreme complexities, substantial variations, and incredible challenges involved in the discharge process, delays have been a common, long-standing, and nationwide problem, which have facilitated increased hospital cost, loss of capacity, bed shortage, more adverse events, and readmissions [2]–[4]. Therefore, improving the quality and efficiency of the discharge process is of significant importance.

The hospital discharge process involves multidisciplinary efforts from multiple care providers, such as physicians, advanced practice providers (APPs) [who are the nurse practitioners (NPs) or physician assistants (PAs)], social workers (SWs), case managers (CMs), occupational therapists (OTs), physical therapists (PTs), pharmacists, and nurses [5]. Substantial efforts have been devoted to reducing delays in the discharge process, mainly focusing on identifying the factors causing the delays. Most of the studies are qualitative or case study-based, which may lack systematic characterization of the discharge process, quantitative analysis of process performance, and in-depth investigation of system behavior. Although analytical models have been widely applied in healthcare systems research, to the best of our knowledge, mathematical models of the discharge process are still unavailable.

In this paper, the complex discharge process at the University of Wisconsin (UW) Hospital is systematically characterized and analyzed by a network with parallel subprocesses, splits, merges, and repeated procedures that comprise the main and unique properties of the system. Then, the network is decomposed into multiple serial processes,

interacting with each other. An analytical method is introduced to evaluate the performance of the serial subprocess and the overall process. Through stochastic modeling of the discharge process, the probability to finish the discharge process within a desired or given time interval, referred to as the discharge-time performance (DTP), as well as the mean and coefficient of variation (CV) of discharge time, can be calculated. Such a model has been validated with high accuracy by simulations and the data collected at UW Hospital. Using this model, the most critical constraining or impeding steps (i.e., bottlenecks) in the discharge process are identified, and various improvement strategies are proposed and evaluated.

The main contribution of this paper lies in characterizing the complex discharge process and workflow, developing an aggregation method for DTP evaluation, and introducing bottleneck analysis for improvement, all from an analytical model perspective, which can provide a fresh and unique view of the system. Comparing with existing work studying serial or assembly processes in manufacturing and healthcare delivery systems, this paper analyzes not only the average output (such as throughput or length of stay) but also the variability and the distribution of discharge time for more complex discharge process with multiple parallel, split, merge, and reentrant features. Such a method can be easily applied to discharge processes at other hospitals and provides a quantitative tool for hospital management to study and improve discharge.

The remainder of this paper is organized as follows. Section II reviews the related literature. The discharge process at UW Hospital is described in Section III. Section IV introduces the analytical model and formulates the problem. The method to evaluate the performance of discharge process is introduced in Section V. Section VI presents the bottleneck analysis and improvement results. Finally, conclusions and future works are summarized in Section VII. All proofs are given in the Appendix.

II. Related Literature

Discharge delays are common in many hospitals. Extensive studies have been carried out to find out the factors leading to delays, such as diagnostic services, medical consultation or investigation, home arrangement, various disruptions, variability in patient's medical condition, and clinical decision-making [6]–[9]. A total of 21 articles on discharge delays have been reviewed in [7], and four types of factors are identified: internal hospital factors, downstream service factors, funding factors, and patient or carer-related factors. In [10], the delays are categorized due to medical or nonmedical factors, internal or external reasons, psychological issues, evaluation errors, hospital capacity limitation, shortage of local facilities, and organizational assessment delays.

To improve the discharge process, different methods, such as appropriateness evaluation protocol, delay tool, and intervention mapping framework, have been used to analyze the length of stay, appropriateness of hospitalization, and causes for delay [11], [12]. Standardization of the process, establishment of guidelines and policies, organizational interventions, timely scheduling of consultation and diagnostic testing, addressing social issues in advance, involving patients and family in discharging planning, and coordination between units and interdisciplinary teams, staff, and patients are advocated [4], [9], [11]–[16]. In [17], 224 articles are reviewed and 47 tools on patient evaluation, planning and teaching, and optimizing discharge summaries are identified.

However, almost all of the above-mentioned studies are qualitative and based on case studies, opinions of expert practitioners or researchers, or standardized lists of criteria [7], which lack quantitative analysis and system property investigation. As there exist considerable heterogeneity and variations due to local contexts [7], [14], identifying the delay factors could be challenging [10]. Thus, a systematic quantitative approach is needed to identify the critical constraints and improve the hospital discharge process.

Analytical models have been widely applied in health care systems to evaluate system performance and facilitate care quality improvement. Bottleneck analysis has been viewed as an effective way for process improvement (see examples in rapid response, diagnosis and test, and surgery flows [18]–[23]). Unfortunately, a few studies have been devoted to analyzing hospital discharge process using a mathematical model. In [5], discrete-event simulations are used to study the hospital discharge process and provide recommendations for potential improvement.

In addition, although Markov and empirical models have been used in manufacturing and healthcare delivery systems (see [24]–[26]), throughput, length of stay, and waiting times are the main average performance measures under investigation, and serial or assembly processes are the main focuses. Only a limited work is devoted to variability study [20], [21], [27]–[30]. Thus, there is a need to develop an analytical model to study the complete distribution of task finishing time in a more complex environment, such as the discharge process. The goal of this paper is contributing to this end.

III. Process Description

UW Hospital is a 505-bed regional referral center that is home to a Level 1 adult and pediatric trauma center. It has an American College of Surgeons verified burn center, one of the nation's largest organ transplant programs, one of the nation's first certified comprehensive stroke centers, and UW Carbone Cancer Center, one of the National Cancer Institute designated comprehensive cancer centers.

A. Discharge Process

The discharge process in the UW Hospital is a multidisciplinary coordinated process, including many subprocesses, components, and numerous variations [5]. The process can vary substantially among different hospitals and different units within a hospital. Even within the same unit, the process can differ depending on many factors, such as physician's preferences, patient conditions, post-acute facilities, and transportation options. In this paper, we focus on the medical unit of the hospital. Based on extensive observations, interviews, and discussions with the discharge team, the discharge process can be described as follows.

The discharge process in a broad sense begins right after the admission of a patient. Upon admission, the SW and CS start to gather information related to the patient's health, insurance, and contacts, where the person previously lived, social supports, and other determinants of social wellbeing, and compile them into a discharge plan of care, which should be almost finished before the final day for discharge. Usually, the discharge signal is given during the morning round meeting (at 9:00 A.M.), which includes physicians (MD), APPs (i.e., NPs or PA), nurses (RN), pharmacists (RPH), SWs, CMs, OT, and PT, depending on the conditions of the patient. During this meeting, a list of patients who are ready for discharge will be informed by the physician, and their conditions and progresses will be reported and discussed by the participating staff. After the meeting, a discharge order is entered into the medical record by the physician and the staff begins preparing for discharge. Many times the APP may orchestrate the discharge process once the MD is off the unit. In this paper, we mainly focus on the process after the morning rounds.

There are mainly three primary parallel subprocesses, represented by the SW/CM workflow, the transportation flow, and the pharmacist workflow, as shown in Fig. 1. A patient cannot be discharged until all the three parallel and independent streams are finished. Typically, even after finishing each process, there could be delays before the final discharge due to waiting for a therapy evaluation and the result of the patient's unawareness. Such a waiting is characterized by "waiting for others" in Fig. 1. The detailed workflows of each subprocess are described next.

Remark 1: Although the discharge processes vary significantly among different hospitals, units, and provider preferences, the basic functionality of the physician, pharmacist, CM, SW, and nurses is similar. Thus, the method introduced in this paper can be applied in other discharge processes with minor changes.

B. SW/CM Workflow

The SW and CM have similar functions and they often work together to complete the contact and confirmation work. Thus, their workflows are modeled together as one. The main work of SW and CM is to communicate and engage with the patient and his/her family regarding the discharge location, aftercare (post-acute) facilities, transportation arrangement, insurance information, and medical record, and finally compile the discharge plan of care. In most cases, such a work is started upon the patient's admission and is considered complete when the patient is discharged. On the day of discharge, the SW and CM only need to have a quick scan to reconfirm all the information and procedure and then prepare the summary report. In the rare case of "unexpected discharge," the SW and CM will meet again with the patient and family, the physicians, and the staff at the post-acute care facility if necessary. In the medical unit of UW Hospital, the percentage of unexpected discharge is estimated to be less than 2%.

The SW and CM workflow is illustrated as the top flow in Fig. 1. The major steps include the following.

1) First, the patient's medical record will be reviewed. If there is no unexpected change, the SW or CM needs to talk to the patient and reconfirm all the information, including aftercare location of facility, insurance, and transportation.
2) If there is any change to be made, the SW or CM will communicate with the physician and talk to the patient as well as his or her family to address all the related issues.
3) If the discharge location is the patient's home, the After Hospital Care Plan will be completed by the interdisciplinary staff, printed by the Health Unit Coordinator of the unit. The nurse will review this with the patient/family prior to discharge.
4) If the discharge location is an aftercare facility, the SW or CM needs to confirm with the facility, review the medical record, and talk to the patient. Finally, they will prepare the discharge packet and necessary documents for the aftercare facility.

To simplify the notation, such steps are referred to as Steps 1–17 of SW/CM flow in Fig. 1. Note that such numbers are used for notation simplification purpose and do not represent any sequence in the process. Similar notations are used for other processes as well.

C. Transportation Workflow

The transportation work characterizes the transportation arrangement, which refers to the period from the discharge decision being made to the time the expected transportation being ready. All patients will need some type of ride to home or to a post-acute care facility. The transportation method and time are set up by the SW/CM right after the discharge decision under the agreement with the patient and physician. If an ambulance is needed for transporting, it may need to be set up the day before discharge. It is observed that only 14% of the patients may need transportation arrangement from UW Hospital. In other times, the transportation workflow can be ignored. Such a process is illustrated in the middle part of Fig. 1 and referred to as Step 1 of transportation (TR) flow.

D. RPH Workflow

The pharmacist workflow is the most critical and complicated one during the day of discharge. The main responsibility of a pharmacist is to check and clear any potential issues in the medical order and communicate with the patient to educate about the prescription after the discharge decision. The specific steps in the workflow include the following.

1) The pharmacist needs to go over the medical order after the morning round meeting. If no error or issue is found, the process can continue without contacting the physician.
2) If there is any issue or question in the medical order, which is about 60% of chances in the hospital, a pharmacist intervention is needed. In this case, the pharmacist will contact the physician to make any necessary changes under the agreement, and the MD/APP has to reorder the medication.

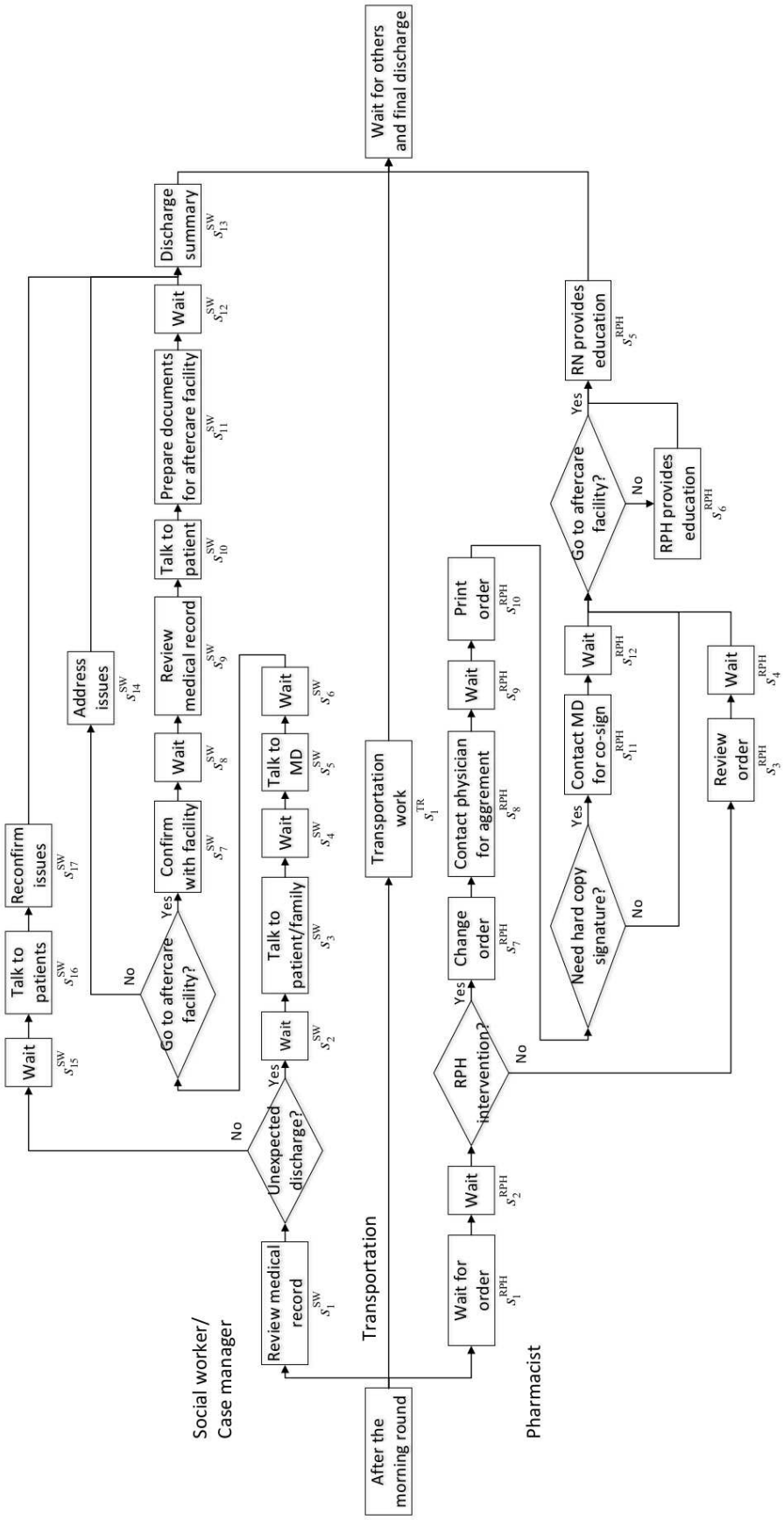

Fig. 1. UW Hospital discharge process.

3) If the home is the discharge location, the pharmacist needs to print out the order and bring to the patient for education. If the aftercare facility is the location, the order will be included in the discharge packet to be sent to the facility.
4) Finally, nurse education is provided to the patient. The nurse is also responsible for getting the supplies or providing further education on something the patient may need.

These steps are illustrated in bottom of Fig. 1 and denoted as Steps 1–12 in the RPH flow.

IV. System Modeling and Problem Formulation

As shown in Fig. 1, the hospital discharge process can be characterized as a complex network with multiple splits, merges, reworks, and parallel lanes. To evaluate the performance of discharge of such a process, introduce the following assumptions.

1) The three parallel workflows are independent. However, the discharge process cannot continue until all SW, transportation, and RPH workflows are finished.
2) The service time (or waiting time) at each step s_i^l is defined by a random process with mean τ_i^l and CV cv_i^l, where i is the step number as shown in Fig. 1 and l represents the workflows of SW/SW, pharmacist (RPH), and transportation (TR). All successive steps are assumed to be independent.
3) In the SW workflow, the probability of unexpected discharge is defined by probability α_{unexp}, which characterizes the possibility that the preparation for discharge is not finished by the discharge date.
4) A patient has probability α_{care} to be discharged to an aftercare facility. In other words, the probability of discharging to home is $1-\alpha_{\mathrm{care}}$.
5) There exists a probability α_{tr} that a patient will require transportation arrangement.
6) In the RPH workflow, a physician's order has probability α_{int} needing a pharmacist's intervention. Among them, there exists a probability α_{sign} that the pharmacist and physician need to co-sign the updated prescription.

Remark 2: In hospitals, usually the resources, such as physicians, nurses, and pharmacists, are limited and need to take responsibility for multiple tasks. Thus, the patients may need to wait before receiving the service for discharge because of availability issue. In this paper, the availabilities of the resources are considered by including the steps named "wait" in the discharge process (as shown in Fig. 1), which represent the time that the patient needs to wait before the next service step due to the unavailability of the resource. In the future work, we plan to include the multiple tasks of providers in the model so that the availability and the resulting waiting time can be generated and evaluated from the model directly.

Remark 3: Based on observations in the hospital, the steps with long durations include transportation work, wait for order, talk to patients, review of medical records, discharge summary, and nurse education. The durations of such steps are mainly determined by the services in their own tasks rather than other steps. Thus, the independence assumption of successive steps is practical.

Under assumptions 1–6, define the time to finish each process as T^l, $l = \mathrm{SW}, \mathrm{TR}, \mathrm{RPH}$. Then, the time to finish the overall discharge process, T, is characterized by

$$T = \max\{T^{\mathrm{SW}}, T^{\mathrm{TR}}, T^{\mathrm{RPH}}\}. \tag{1}$$

Note that here T does not include the last waiting time ("waiting for others") in Fig. 1 since such a step may not be observed in every discharge process. Clearly, such a time can be easily included by addition.

Introduce the discharge-time performance (DTP) as the probability to finish the discharge process within a desired or given time interval T_{given}. Then, DTP is a function of all system variables, i.e.,

$$\mathrm{DTP}(T_{\mathrm{given}}) = \Pr(T \le T_{\mathrm{given}}) = f(\mathbf{M}, \mathbf{V}, \mathbf{A}, T_{\mathrm{given}}) \tag{2}$$

where

$$\begin{aligned}\mathbf{M} &= \left[\tau_1^{\mathrm{SW}}, \ldots, \tau_{17}^{\mathrm{SW}}, \tau_1^{\mathrm{TR}}, \tau_1^{\mathrm{RPH}}, \ldots, \tau_{12}^{\mathrm{RPH}}\right]\\ \mathbf{V} &= \left[cv_1^{\mathrm{SW}}, \ldots, cv_{17}^{\mathrm{SW}}, cv_1^{\mathrm{TR}}, cv_1^{\mathrm{RPH}}, \ldots, cv_{12}^{\mathrm{RPH}}\right]\\ \mathbf{A} &= \left[\alpha_{\mathrm{unexp}}, \alpha_{\mathrm{care}}, \alpha_{\mathrm{tr}}, \alpha_{\mathrm{int}}, \alpha_{\mathrm{sign}}\right].\end{aligned}$$

Due to the independence assumption of parallel lanes, $\mathrm{DTP}(T_{\mathrm{given}})$ can be evaluated as

$$\begin{aligned}\mathrm{DTP}(T_{\mathrm{given}}) &= \Pr(T^{\mathrm{SW}} \le T_{\mathrm{given}}) \cdot \Pr(T^{\mathrm{TR}} \le T_{\mathrm{given}})\\ &\quad \cdot \Pr(T^{\mathrm{RPH}} \le T_{\mathrm{given}})\\ &= \prod_{l=\mathrm{SW,TR,RPH}} \mathrm{DTP}^l(T_{\mathrm{given}})\end{aligned} \tag{3}$$

where $\mathrm{DTP}^l(T_{\mathrm{given}})$ indicates the probability to finish workflow l within time interval T_{given}.

When DTP is obtained, the expected discharge time T_d can be calculated. Thus

$$T_d = E(T) = \int_0^\infty (1 - \mathrm{DTP}(x))dx. \tag{4}$$

In addition to the average value, the variability of discharge time, characterized by the CV, CV_d can be evaluated as

$$CV_d = \frac{\sqrt{Var(T)}}{E(T)} = \frac{\sqrt{E[(T-T_d)^2]}}{T_d}. \tag{5}$$

Then, the problem to be studied can be formulated as follows. Under assumptions 1–6, develop a method to calculate DTP, T_d, and CV_d and investigate improvement strategies.

Solutions to the problem are provided in Sections V and VI.

V. Evaluation of Discharge-Time Performance

A. Process Decomposition

As shown in Fig. 1 and assumptions 1–6, the discharge process includes multiple parallel subprocesses, splits, merges, and reworks with random variables following general distributions. The direct analysis of the process performance is all but impossible. In fact, even the mean and variance of discharge time are difficult to calculate. Evaluating the discharge-time performance (in other words, the distribution of discharge time) will be more challenging.

However, considering that each patient can only take one specific procedure or route in SW/CM workflow, transportation, and RPH subprocesses, we can decompose the discharge process into a collection of serial procedures, each with a certain probability. Specifically, in the SW/CM workflow, the following serial procedures are possible.

1) SW_1: Unexpected discharge occurs, and the patient is discharged to aftercare facility.
2) SW_2: Unexpected discharge occurs, and the patient is discharge to home.
3) SW_3: No unexpected discharge occurs.

For transportation flow, there are only two scenarios as follows.

1) TR_1: Need transportation work.
2) TR_2: No need for transportation work.

The RHP workflow involves more possibilities as follows.

1) RPH_1: Need RPH intervention, need hard copy signature, and the patient is discharged to aftercare facility.
2) RPH_2: Need RPH intervention, need hard copy signature, and the patient is discharged to home.
3) RPH_3: Need RPH intervention, do not need hard copy signature, and the patient is discharged to aftercare facility.
4) RPH_4: Need RPH intervention, do not need hard copy signature, and the patient is discharged to home.
5) RPH_5: Do not need RPH intervention, and the patient is discharged to aftercare facility.
6) RPH_6: Do not need RPH intervention, and the patient is discharged to home.

An illustration of these serial procedures is presented in Fig. 2. Then, the probability for each serial procedure can be denoted as

$$\mathbf{P} = \left[p_1^{\mathrm{SW}}, \ldots, p_3^{\mathrm{SW}}, p_1^{\mathrm{TR}}, p_2^{\mathrm{TR}}, p_1^{\mathrm{RPH}}, \ldots, p_6^{\mathrm{RPH}}\right] \quad (6)$$

where the subscript corresponds to the procedure number defined in each workflow and

$$\begin{aligned}
p_1^{\mathrm{SW}} &= \alpha_{\mathrm{unexp}}\alpha_{\mathrm{care}} \\
p_2^{\mathrm{SW}} &= \alpha_{\mathrm{unexp}}(1-\alpha_{\mathrm{care}}) \\
p_3^{\mathrm{SW}} &= 1-\alpha_{\mathrm{unexp}} \\
p_1^{\mathrm{TR}} &= \alpha_{\mathrm{tr}} \\
p_2^{\mathrm{TR}} &= 1-\alpha_{\mathrm{tr}} \\
p_1^{\mathrm{RPH}} &= \alpha_{\mathrm{int}}\alpha_{\mathrm{sign}}\alpha_{\mathrm{care}} \\
p_2^{\mathrm{RPH}} &= \alpha_{\mathrm{int}}\alpha_{\mathrm{sign}}(1-\alpha_{\mathrm{care}}) \\
p_3^{\mathrm{RPH}} &= \alpha_{\mathrm{int}}(1-\alpha_{\mathrm{sign}})(1-\alpha_{\mathrm{care}}) \\
p_5^{\mathrm{RPH}} &= (1-\alpha_{\mathrm{int}})\alpha_{\mathrm{care}} \\
p_6^{\mathrm{RPH}} &= (1-\alpha_{\mathrm{int}})(1-\alpha_{\mathrm{care}}). \quad (7)
\end{aligned}$$

In addition, if n^l is the number of decomposed serial procedures in workflow l described earlier, then

$$n^{\mathrm{SW}} = 3, \quad n^{\mathrm{TR}} = 2, \quad n^{\mathrm{RPH}} = 6.$$

In addition

$$\sum_{k=1}^{n^l} p_k^l = 1, \quad l = \mathrm{SW}, \mathrm{TR}, \mathrm{RPH}. \quad (8)$$

Using such a decomposition method, the complex discharge process can be represented by a group of serial procedures, and each of them consists of multiple sequential steps. For each patient, his/her discharge process is the combination of three parallel procedures, each being selected from the sets of SW/CM workflow, transportation flow, and RPH workflow. In each flow, we obtain the total time as

$$T^l = \frac{\sum_{k=1}^{n^l} p_k^l T_k^l}{\sum_{k=1}^{n^l} p_k^l}, \quad l = \mathrm{SW}, \mathrm{TR}, \mathrm{RPH} \quad (9)$$

where T_k^l is the time spent in procedure k of workflow l. When T_k^l and T^l are known, the discharge time T can be calculated from (1), and the discharge-time performance DTP can be evaluated from (2).

B. Performance Evaluation

However, even if the complex discharge process can be decomposed into a set of serial procedures, an analysis of the finishing time in each procedure is still difficult. Since each procedure will be the sum of multiple random variables following arbitrary distributions, the calculation of DTP is not straightforward, and the evaluation of mean and CV of discharge time relies on DTP calculation. To solve this problem, we propose to approximate each service or step using a gamma distribution based on the first two moments. As it has been shown that in many healthcare systems [20]–[23], [31]–[35] and manufacturing systems [24], when the CVs of the processes are small (less than 1), the overall performance could be primarily dependent on the first two moments of the process rather than the complete distribution. Since a gamma distribution depends on two parameters that enable us to place the mean and variance with much freedom, it can represent many distributions with significantly varied shapes. In addition, it is a fairly flexible positive-skewed distribution with convenient mathematical properties, which enables us to carry out more complex analytical investigation. However, this is only suitable when the variability of the process is small (i.e., CV < 1). Thus, we hypothesize that a gamma distribution can be used to approximate the discharge process.

Specifically, denote the mean and CV of the jth service or waiting time in serial procedure k of workflow l as $\tau_{k,(j)}^l$ and $\mathrm{cv}_{k,(j)}^l$, respectively. In addition, introduce n_k^l as the number of steps in such a procedure (i.e., procedure k of workflow l). Then, we have

$$\begin{aligned}
T_k^l &= \sum_{j=1}^{n_k^l} \tau_{k,(j)}^l, \quad l = \mathrm{SW}, \mathrm{TR}, \mathrm{RPH} \\
CV_k^l &= \frac{\sqrt{\sum_{j=1}^{n_k^l} \left(cv_{k,(j)}^l \cdot \tau_{k,(j)}^l\right)^2}}{T_k^l}, \quad l = \mathrm{SW}, \mathrm{TR}, \mathrm{RPH}. \quad (10)
\end{aligned}$$

By assuming that each step follows a gamma distribution, it is equivalent to evaluate the cumulative distribution function (cdf) of the sum of independently distributed gamma

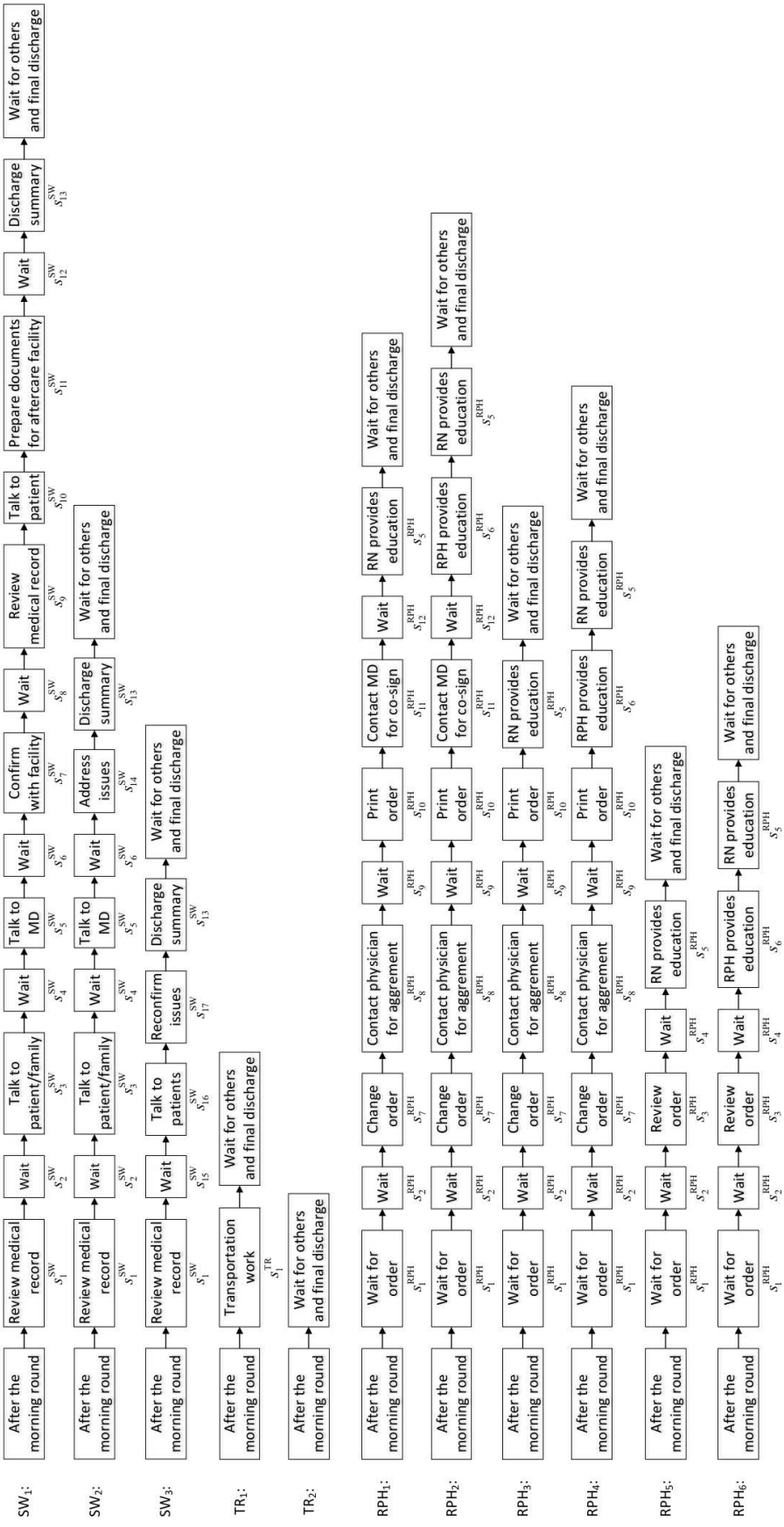

Fig. 2. Decomposed serial procedures.

TABLE I

MEAN AND CV OF SERVICE/WAITING TIME OF EACH STEP AT UW HOSPITAL. (a) MEAN SERVICE/WAITING TIME. (b) CV OF SERVICE/WAITING TIME

τ_1^{SW}	τ_2^{SW}	τ_3^{SW}	τ_4^{SW}	τ_5^{SW}	τ_6^{SW}	τ_7^{SW}	τ_8^{SW}	τ_9^{SW}	τ_{10}^{SW}
5	2	4	10	1	1	8	11	11	11
τ_{11}^{SW}	τ_{12}^{SW}	τ_{13}^{SW}	τ_{14}^{SW}	τ_{15}^{SW}	τ_{16}^{SW}	τ_{17}^{SW}	τ_1^{TR}	τ_1^{RPH}	τ_2^{RPH}
7	8	11	5	2	5	5	233	217	2
τ_3^{RPH}	τ_4^{RPH}	τ_5^{RPH}	τ_6^{RPH}	τ_7^{RPH}	τ_8^{RPH}	τ_9^{RPH}	τ_{10}^{RPH}	τ_{11}^{RPH}	τ_{12}^{RPH}
8	1	11	4	8	1	20	2	1	20

(a)

cv_1^{SW}	cv_2^{SW}	cv_3^{SW}	cv_4^{SW}	cv_5^{SW}	cv_6^{SW}	cv_7^{SW}	cv_8^{SW}	cv_9^{SW}	cv_{10}^{SW}
0.31	0.29	0.47	0.43	0.12	0.18	0.33	0.42	0.36	0.35
cv_{11}^{SW}	cv_{12}^{SW}	cv_{13}^{SW}	cv_{14}^{SW}	cv_{15}^{SW}	cv_{16}^{SW}	cv_{17}^{SW}	cv_1^{TR}	cv_1^{RPH}	cv_2^{RPH}
0.16	0.40	0.18	0.12	0.21	0.40	0.19	0.84	0.67	0.23
cv_3^{RPH}	cv_4^{RPH}	cv_5^{RPH}	cv_6^{RPH}	cv_7^{RPH}	cv_8^{RPH}	cv_9^{RPH}	cv_{10}^{RPH}	cv_{11}^{RPH}	cv_{12}^{RPH}
0.19	0.25	0.25	0.19	0.19	0.41	0.50	0.12	0.29	0.47

(b)

variables. If n^l is the number of serial procedures of workflow l, then the discharge-time performance can be calculated as follows.

Proposition 1: Under assumptions 1–6, if the service/waiting time at each step follows a gamma distribution, the discharge-time performance can be calculated as

$$\mathrm{DTP}(T_{\mathrm{given}}) = \prod_{l=\mathrm{SW,TR,RPH}} \frac{\sum_{k=1}^{n^l} p_k^l G_k^l(T_{\mathrm{given}})}{\sum_{k=1}^{n^l} p_k^l} \tag{11}$$

where $G_k^l(T_{\mathrm{given}})$ represents the cdf of serial procedure k in workflow l, i.e., the probability the time spent in this procedure is less than T_{given}, which can be evaluated as

$$G_k^l(T_{\mathrm{given}}) = \prod_{j=1}^{n_k^l} \left(\frac{\beta_{k,\min}^l}{\beta_{k,(j)}^l}\right)^{\eta_{k,(j)}^l} \cdot \sum_{m=0}^{\infty} \frac{\delta_{k,m}^l \gamma\left(\rho_k^l + m, \frac{T_{\mathrm{given}}}{\beta_{k,\min}^l}\right)}{\Gamma\left(\rho_k^l + m\right)} \tag{12}$$

and

$$\begin{aligned}
\eta_{k,(j)}^l &= \frac{1}{\left(cv_{k,(j)}^l\right)^2}, \quad \beta_{k,(j)}^l = \left(cv_{k,(j)}^l\right)^2 \cdot \tau_{k,(j)}^l \\
\beta_{k,\min}^l &= \min\left(\beta_{k,(j)}^l\right), \quad j = 1, \ldots, n_k^l \\
\rho_k^l &= \sum_{j=1}^{n_k^l} \eta_{k,(j)}^l, \quad \delta_{k,0}^l = 1, \\
\upsilon_{k,m}^l &= \frac{1}{m} \sum_{j=1}^{n_j^l} \eta_{k,(j)}^l \left(1 - \frac{\beta_{k,\min}^l}{\beta_{k,(j)}^l}\right)^m, \quad m = 1, 2, \ldots \\
\delta_{k,m}^l &= \frac{1}{m+1} \sum_{j=1}^{m+1} j \upsilon_{k,j} \delta_{k,m+1-j}, \quad m = 1, 2, \ldots \\
\gamma(a, x) &= \int_0^x y^{a-1} e^{-y} dy \\
\Gamma(a) &= \lim_{x \to \infty} \gamma(a, x).
\end{aligned} \tag{13}$$

Proof: See the Appendix.

Using $\mathrm{DTP}(T_{\mathrm{given}})$, the mean and CV of discharge times, T_d and CV_d, can be evaluated from (4) and (5).

C. Model Validation

Proposition 1 introduces an approximation method to calculate discharge-time performance. To validate the model, two questions arise. First, when the variability of each step is small (i.e., its CV < 1), is the discharge-time performance mainly dependent on the mean and CV of each step rather than the complete distribution? In other words, is it acceptable to use gamma distribution to characterize each random variable? As explained earlier, the reason to use gamma distribution is due to the large freedom of placing mean and variance and its good analytical property. If the answer is positive, then the next question is related to the accuracy of such approximation, i.e., can Proposition 1 provide a precise estimate of discharge-time performance?

First, we investigate the feasibility of gamma approximation. Two simulation models are compared. One assumes that every step is represented by a gamma distribution, while the other assumes that each step follows a randomly selected distribution (either gamma, lognormal, Weibull, or a mixture of them). However, the same first two moments are assumed in both scenarios at each step. Using the data collected at UW Hospital (see Table I), we randomly generate mean service time at each step between 50% and 150% of the mean value in Table I and randomly select the CV between 0.1 and 1.0. The simulations are setup as follows; 10 000 patients are simulated to collect data and 20 replications are executed to ensure the confidence interval small enough to be less than 1% of the DTP values. Then, the discharge-time performances are simulated and compared for 20 data sets. The results are shown in Table II, where the average, minimum, and maximum differences between the two models for given T_{given} values are presented.

TABLE II
DTP DIFFERENCES IN DISTRIBUTION TYPES

T_{given}	Ave. diff.	Min. diff.	Max. diff.
20	0.000031	0.000000	0.001000
40	0.001375	0.000000	0.016200
60	0.004029	0.000000	0.028700
80	0.006223	0.000000	0.041600
100	0.007323	0.000000	0.036000
120	0.007554	0.000000	0.037600
140	0.007082	0.000000	0.032900
160	0.006314	0.000000	0.033000
180	0.006647	0.000000	0.024300
200	0.006681	0.000000	0.025500
220	0.005831	0.000100	0.024500
240	0.006229	0.000000	0.027400
260	0.005963	0.000000	0.021800
280	0.006476	0.000000	0.026700
300	0.006445	0.000000	0.027200
320	0.005986	0.000000	0.023600
340	0.005914	0.000000	0.028500
360	0.005710	0.000000	0.023500
380	0.005198	0.000000	0.026200
400	0.004858	0.000000	0.025000
420	0.004504	0.000000	0.025800
440	0.004023	0.000000	0.021100
460	0.003787	0.000000	0.023000
480	0.003531	0.000000	0.024800
500	0.003309	0.000000	0.018600
520	0.002761	0.000000	0.019100
540	0.002643	0.000000	0.022800
560	0.002297	0.000000	0.019500
580	0.002042	0.000000	0.018100
600	0.001969	0.000000	0.013800
620	0.001594	0.000000	0.018900
640	0.001591	0.000000	0.012000
660	0.001324	0.000000	0.009900
680	0.001214	0.000000	0.012500
700	0.001182	0.000000	0.007900
720	0.000939	0.000000	0.006700
740	0.001021	0.000000	0.008700
760	0.000894	0.000000	0.005500
780	0.000831	0.000000	0.007800
800	0.000767	0.000000	0.007500

TABLE III
DTP COMPARISON WITH SIMULATIONS

T_{given}	Ave. diff.	Min. diff.	Max. diff.
20	0.000074	0.000000	0.000481
40	0.002441	0.000006	0.009920
60	0.006311	0.000043	0.020637
80	0.009557	0.000039	0.029293
100	0.011568	0.000110	0.038379
120	0.012637	0.000145	0.033905
140	0.012466	0.000061	0.032920
160	0.011214	0.000056	0.030747
180	0.009863	0.000098	0.029598
200	0.007597	0.000099	0.028925
220	0.006079	0.000010	0.016512
240	0.005180	0.000038	0.015162
260	0.003897	0.000009	0.016195
280	0.003556	0.000040	0.011235
300	0.003792	0.000084	0.015784
320	0.004309	0.000081	0.019981
340	0.005041	0.000317	0.023570
360	0.005592	0.000006	0.020238
380	0.006215	0.000009	0.021478
400	0.005915	0.000078	0.022321
420	0.005839	0.000069	0.024413
440	0.005515	0.000040	0.023599
460	0.005493	0.000040	0.019925
480	0.005361	0.000018	0.021037
500	0.005278	0.000013	0.018283
520	0.005514	0.000025	0.022211
540	0.005064	0.000005	0.019772
560	0.004638	0.000036	0.018117
580	0.004422	0.000013	0.019200
600	0.004355	0.000012	0.018375
620	0.003594	0.000008	0.014699
640	0.003864	0.000011	0.017131
660	0.002996	0.000011	0.014531
680	0.003630	0.000003	0.017461
700	0.002760	0.000004	0.011885
720	0.002720	0.000030	0.013568
740	0.002593	0.000033	0.013579
760	0.002549	0.000016	0.012429
780	0.002292	0.000020	0.012360
800	0.001983	0.000024	0.010572

As one can see, the differences are very small, which implies that the discharge-time performance is practically independent of distribution type but mainly depends on the mean and CV of the service or waiting time. This justifies that the gamma distribution can be used to characterize the service or waiting time at each step. Fig. 3 presents four examples by comparing the model having gamma distribution at each step with the one having a randomly selected distribution of lognormal, Weibull, or gamma (referred to as "mixed" in the figure) at each step. Again, the differences are minimal. Thus, a "distribution-free" property can be observed (by ignoring the minor differences).

When the "distribution-free" property is justified, we next use gamma distribution to characterize the service/waiting time at each step and then apply Proposition 1 to calculate DTP. The results are compared with that from the simulation model by randomly selecting distributions. The average, minimum, and maximum differences are shown in Table III, which are very small. To illustrate, four examples are presented in Fig. 4.

Moreover, the data collected from about 3000 discharge records and 50 observations at UW Hospital (Table I) are used for comparison as well. First, we calculate the average discharge time as 270.35 min with the CV being 0.53. While the observed data show an average discharge time of 276.48 min and a CV of 0.38. Second, the discharge-time performance calculated from Proposition 1 is compared with simulation assuming randomly selected distributions at each step. The results are shown in Fig. 5. In all scenarios, the estimations have an acceptable accuracy.

Since T_d and CV_d are dependent on $\text{DTP}(T_{\text{given}})$, their accuracies are similar to those of DTP.

VI. IMPROVEMENT ANALYSIS

Using the performance evaluation method introduced earlier, we seek to improve the efficacy of discharge process. Particularly, identifying the most critical step and factors impeding the discharge process and mitigating their effects are of importance. To do this, monotonicity is investigated first to identify the directions of potential improvement.

A. Monotonicity

Proposition 2: Under assumptions 1–6, the discharge-time performance is monotonically decreasing with respect to the mean waiting time of each step.

Proof: See the Appendix.

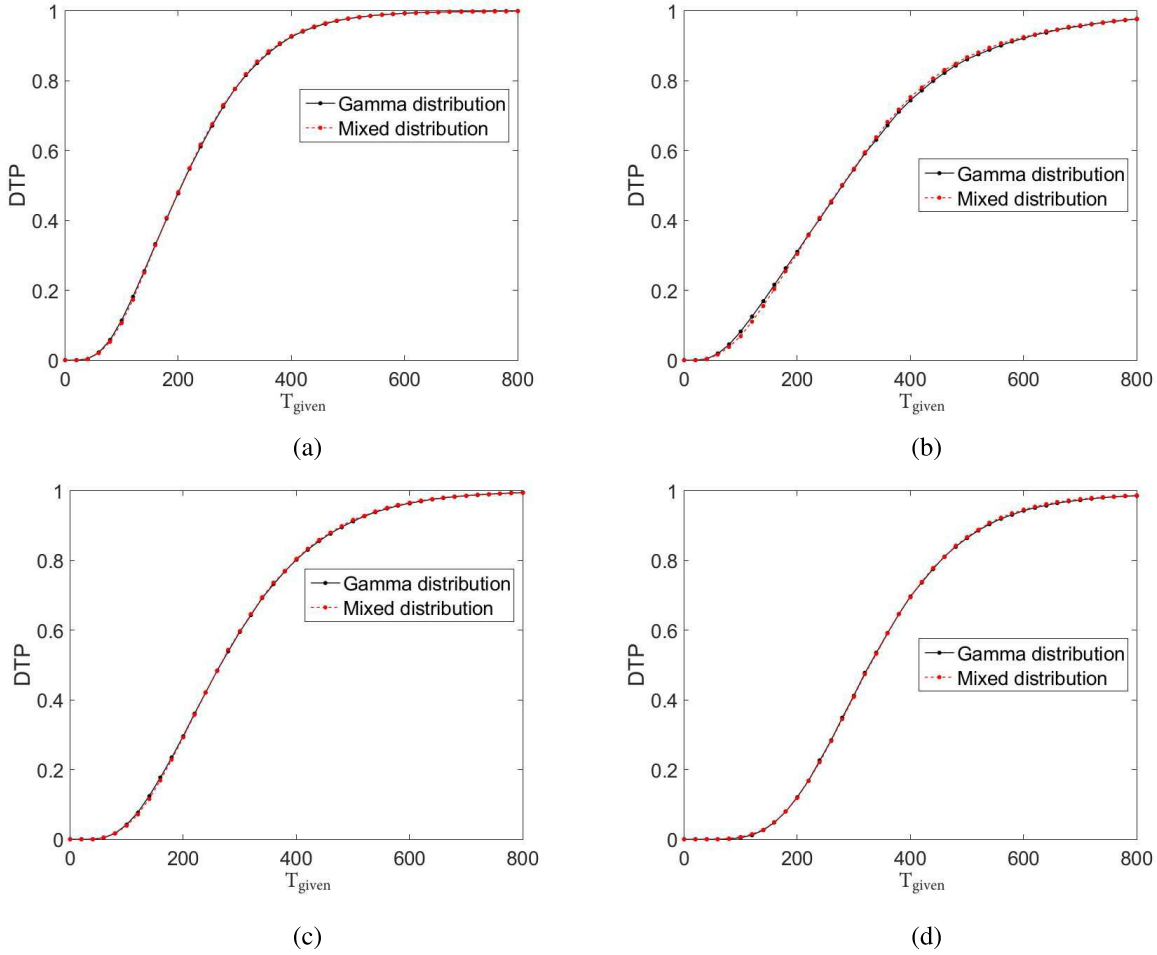

Fig. 3. Comparison examples in distribution type. (a) Example 1. (b) Example 2. (c) Example 3. (d) Example 4.

Such a property is also verified through extensive simulations and numerical experiments. As shown in Fig. 6(a), when the mean service/waiting time is reduced by 10%, the DTP is always increased. However, such increases are not the same for different values of τ_i^l. In Fig. 6(a), the corresponding DTP curves due to reduction in τ_1^{SW}, τ_{13}^{SW}, τ_1^{TR}, τ_1^{RPH}, and τ_5^{RPH} are illustrated. As one can see, a small reduction in waiting time at step s_1^{RPH} of RPH workflow (i.e., waiting for physician discharge order) will lead to much more increase in DTP comparing the reduction in other steps, which only result in a very small increase in DTP.

Typically, the upper portion of DTP or larger T_{given} is where the interest is in. When T_{given} is large, as shown in Fig. 6(b), when the CV of service/waiting time is reduced by 20%, the DTP is increased. As the mean and CV of total time in each procedure are monotonically decreasing when those of service/waiting time at each step are decreasing, the resulting total discharge time will decrease as well, but the DTP is increasing. Again, reducing CV of waiting time at step s_1^{RPH} of RPH workflow has a much larger increase in DTP comparing with reducing time at other steps, where the DTP increases are very small.

To further illustrate such behaviors, Table IV provides the detailed improvement in DTPs for mean and CV reduction of the service/waiting times at each step. It is clear that reducing τ_1^{RPH} and $\mathrm{cv}_1^{\mathrm{RPH}}$ could result in the largest improvement in DTP.

Similar to DTP monotonicity, the mean discharge time also exhibits monotonic property with respect to the mean service/waiting time at each step.

Proposition 3: Under assumption 1–6, the mean discharge time T_d is monotonically increasing with respect to the mean of waiting time of each step.

Proof: See the Appendix.

Fig. 7 shows such a behavior. As shown in the figure, the expected discharge time is monotonically increasing with respect to the mean time waiting for physician order (Step 1 in the RPH workflow).

Concerning the CV of discharge time, intuitively, it should monotonically increase with respect to the CV of service/waiting time at each step. Such a property is validated through extensive numerical experiments. Three examples are presented in Fig. 8, where the monotonicity with respect to the waiting time for physician order (Step 1 in the RPH workflow), document preparation time for aftercare facility (step 11 in the SW/CM workflow), and transportation time (Step 1 in the TR workflow) is illustrated. It is shown that the CV of discharge time is more sensitive to the CV of waiting time for physician order.

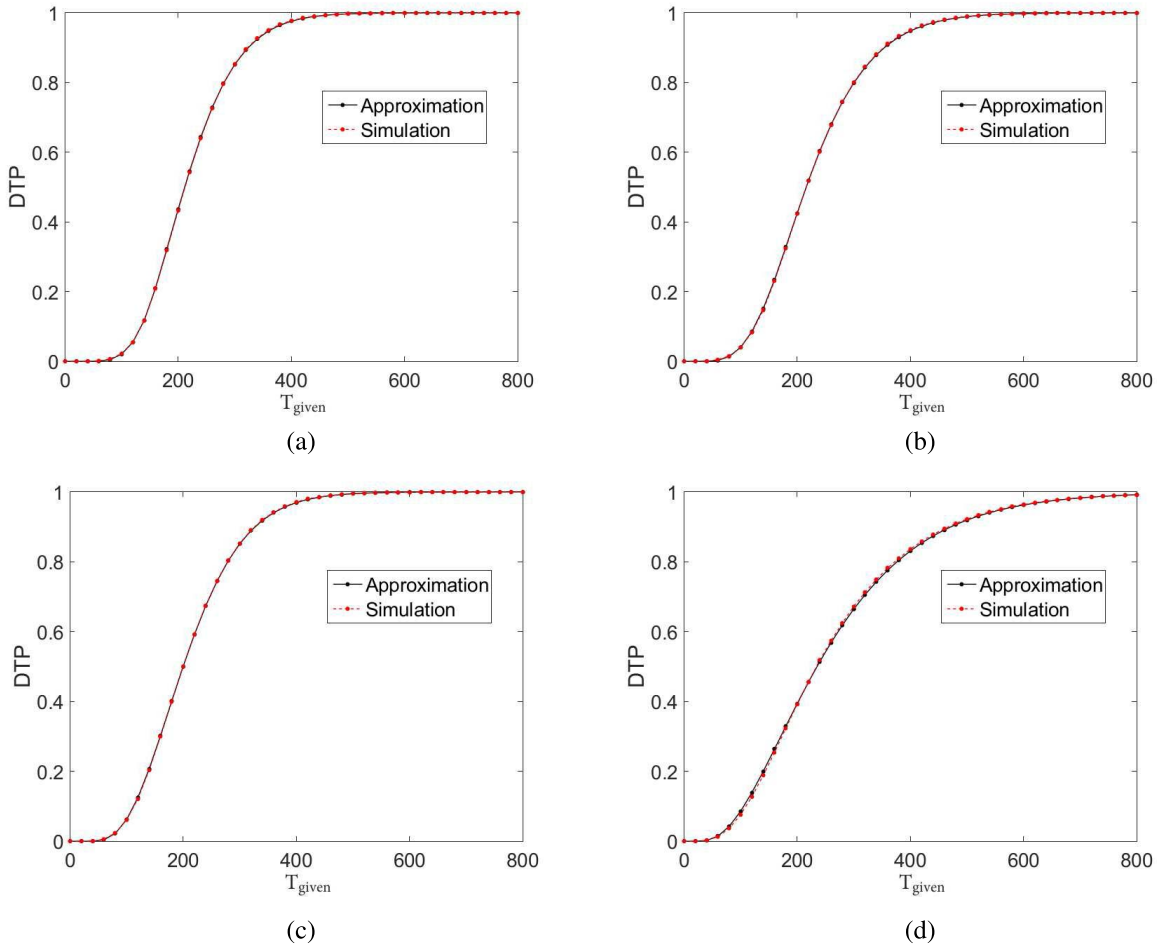

Fig. 4. Comparison examples with simulation. (a) Example 1. (b) Example 2. (c) Example 3. (d) Example 4.

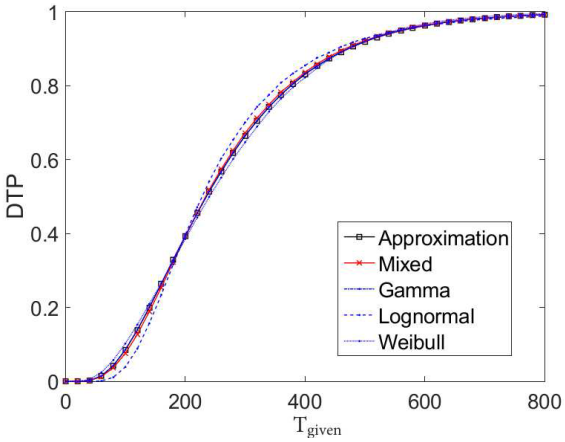

Fig. 5. Comparison example using hospital data.

Remark 4: Numerical experiments indicate that each step's mean time τ has a very small influence on the CV of discharge time, and the mean discharge time is insensitive to the CV of service/waiting time at each step.

B. Bottleneck Analysis

The monotonicity study indicates that reducing each step's τ and CV could lead to higher DTP, but the resulting increase in DTP can be significantly different when different steps are considered. Then, the question arises, which step's reduction could lead to the largest improvement in DTP comparing with reduction in other steps? In other words, which step impedes the discharge-time performance in the strongest manner? We refer such a step to as the bottleneck step. Formally, for a continuous function of DTP, the bottleneck step can be defined as follows.

Definition 1: Step s_i^l is the mean time DTP bottleneck (BN-DTP$_\tau$) if

$$\left|\frac{\partial \mathrm{DTP}}{\partial \tau_i^l}\right| > \left|\frac{\partial \mathrm{DTP}}{\partial \tau_j^k}\right|, \quad \forall \{j,k\} \neq \{i,l\}.$$

Definition 2: Step s_i^l is the variability DTP bottleneck (BN-DTP$_{\mathrm{cv}}$) if

$$\left|\frac{\partial \mathrm{DTP}}{\partial cv_i^l}\right| > \left|\frac{\partial \mathrm{DTP}}{\partial cv_j^k}\right|, \quad \forall \{j,k\} \neq \{i,l\}.$$

However, the partial derivatives are difficult to evaluate analytically. To identify the bottleneck steps, let DTP$(\tau_i^l - \delta_\tau \tau_i^l)$ and DTP$(cv_i^l - \delta_{\mathrm{cv}} cv_i^l)$ be the DTPs when the mean and CV of service/waiting time at step s_i^l are reduced by proportion δ_τ

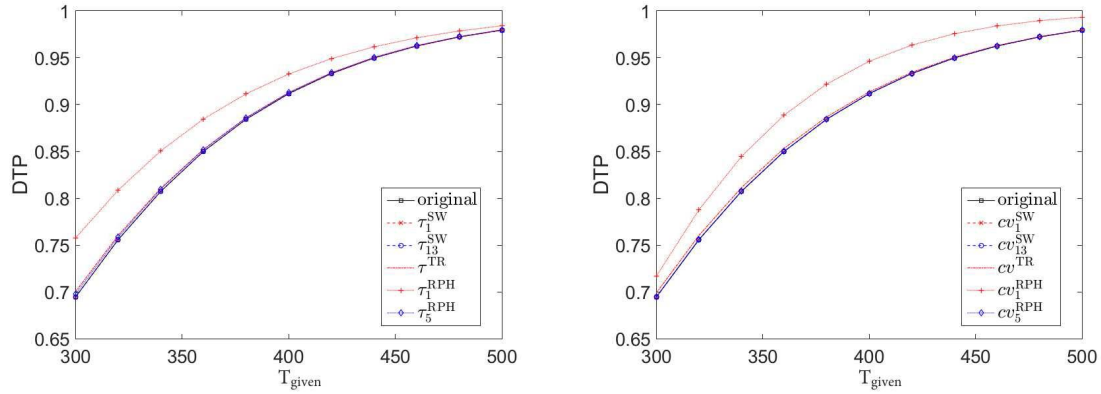

Fig. 6. WTP monotonicity. (a) DTP monotonicity with respect to τ. (b) DTP monotonicity with respect to cv.

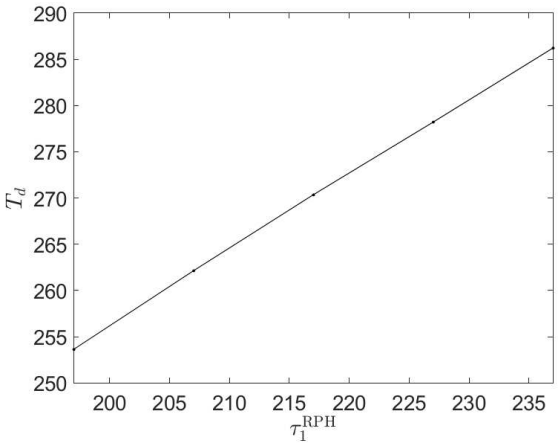

Fig. 7. Mean time monotonicity with respect to τ.

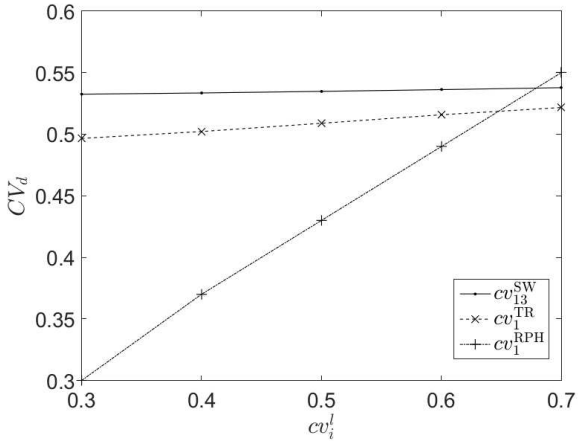

Fig. 8. CV monotonicity with respect to cv.

and δ_{cv}, respectively. In addition, $\delta_\tau \ll 1$ and $\delta_{\text{cv}} \ll 1$. Then, the following approach is used to identify the bottlenecks.

1) Mean time τ_i^l is the BN-DTP$_\tau$ if

$$\text{DTP}\left(\tau_i^l - \delta_\tau \tau_i^l\right) > \text{DTP}\left(\tau_j^k - \delta_\tau \tau_j^k\right), \quad \forall \{j,k\} \neq \{i,l\}.$$

2) CV cv_i^L is the BN-DTP$_{\text{cv}}$ if

$$\text{DTP}\left(cv_i^l - \delta_{\text{cv}} cv_i^l\right) > \text{DTP}\left(cv_j^k - \delta_{\text{cv}} cv_j^k\right), \quad \forall \{j,k\} \neq \{i,l\}.$$

Using such a method, based on the discharge data listed in Table I, we evaluate DTP improvement at the hospital by assuming $\delta_\tau = 0.1$ and $\delta_{\text{cv}} = 0.2$. The results are presented in Table IV.

As one can see, the reduction of mean or CV of service/waiting time of SW/CM makes insignificant improvement to DTP, due to their short service time and low rate of unexpected discharge. The decrease in τ_1^{RPH} and cv_1^{RPH} leads to the largest improvement in DTP. Thus, the "wait for order" step is the bottleneck. Reducing the waiting time for physician's order will substantially improve the discharge process.

Similarly, the mean time and CV bottlenecks for discharge time can be defined and evaluated. Specifically, it is shown in Definition 3.

Definition 3: Step s_i^l is the mean discharge time bottleneck (BN-τ) if

$$\left|\frac{\partial T_d}{\partial \tau_i^l}\right| > \left|\frac{\partial T_d}{\partial \tau_j^k}\right|, \quad \forall \{j,k\} \neq \{i,l\}.$$

Definition 4: Step s_i^l is the discharge time variability bottleneck (BN-CV) if

$$\left|\frac{\partial CV_d}{\partial cv_i^l}\right| > \left|\frac{\partial CV_d}{\partial cv_j^k}\right|, \quad \forall \{j,k\} \neq \{i,l\}.$$

As the calculations of T_d and CV_d are complicated, we evaluate the differences in mean time and CV reduction to identify the bottlenecks.

1) Mean time τ_i^l is the BN-τ if

$$T_d\left(\tau_i^l - \delta_\tau \tau_i^l\right) > T_d\left(\tau_j^k - \delta_\tau \tau_j^k\right), \quad \forall \{j,k\} \neq \{i,l\}.$$

2) CV cv_i^L is the BN-CV if

$$CV_d\left(cv_i^l - \delta_{\text{cv}} cv_i^l\right) > CV_d\left(cv_j^k - \delta_{\text{cv}} cv_j^k\right) \quad \forall \{j,k\} \neq \{i,l\}.$$

TABLE IV
DTP IMPROVEMENTS WITH RESPECT TO MEAN OR CV REDUCTION. (a) REDUCING τ. (b) REDUCING CV

T_{given}	$\tau_1^{\text{SW}} \sim \tau_{17}^{\text{SW}}$	τ_1^{TR}	τ_1^{RPH}	τ_2^{RPH}	τ_3^{RPH}	τ_4^{RPH}	τ_5^{RPH}
300	0.0000	0.0042	0.0541	0.0004	0.0006	0.0001	0.0022
400	0.0000	0.0041	0.0386	0.0002	0.0003	0.0000	0.0012
500	0.0000	0.0033	0.0229	0.0001	0.0002	0.0000	0.0006
T_{given}	τ_6^{RPH}	τ_7^{RPH}	τ_8^{RPH}	τ_9^{RPH}	τ_{10}^{RPH}	τ_{11}^{RPH}	τ_{12}^{RPH}
300	0.0006	0.0010	0.0001	0.0026	0.0003	0.0000	0.0003
400	0.0003	0.0006	0.0001	0.0015	0.0001	0.0000	0.0002
500	0.0002	0.0003	0.0000	0.0007	0.0001	0.0000	0.0001

(a)

T_{given}	$cv_1^{\text{SW}} \sim cv_{17}^{\text{SW}}$	cv_1^{TR}	cv_1^{RPH}	cv_2^{RPH}	cv_3^{RPH}	cv_4^{RPH}	cv_5^{RPH}
300	0.0000	0.0007	0.0080	0.0000	0.0000	0.0000	0.0000
400	0.0000	0.0034	0.0334	0.0000	0.0000	0.0000	0.0000
500	0.0000	0.0040	0.0290	0.0000	0.0000	0.0000	0.0000
T_{given}	cv_6^{RPH}	cv_7^{RPH}	cv_8^{RPH}	cv_9^{RPH}	cv_{10}^{RPH}	cv_{11}^{RPH}	cv_{12}^{RPH}
300	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000
400	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000
500	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

(b)

TABLE V
IMPROVEMENTS IN MEAN DISCHARGE TIME

	$\tau_1^{\text{SW}} \sim \tau_{17}^{\text{SW}}$	τ_1^{TR}	τ_1^{RPH}	τ_2^{RPH}	τ_3^{RPH}	τ_4^{RPH}	τ_5^{RPH}
ΔT_d	0.00	1.12	18.48	0.18	0.29	0.04	0.99
	τ_6^{RPH}	τ_7^{RPH}	τ_8^{RPH}	τ_9^{RPH}	τ_{10}^{RPH}	τ_{11}^{RPH}	τ_{12}^{RPH}
ΔT_d	0.27	0.43	0.05	1.08	0.12	0.01	0.11

TABLE VI
IMPROVEMENTS IN THE CV OF DISCHARGE TIME

	$\tau_1^{\text{SW}} \sim \tau_{17}^{\text{SW}}$	τ_1^{TR}	τ_1^{RPH}	τ_2^{RPH}	τ_3^{RPH}	τ_4^{RPH}	τ_5^{RPH}
ΔCV_d	0.0000	0.0035	0.0420	0.0000	0.0000	0.0000	0.0000
	cv_6^{RPH}	cv_7^{RPH}	cv_8^{RPH}	cv_9^{RPH}	cv_{10}^{RPH}	cv_{11}^{RPH}	cv_{12}^{RPH}
ΔCV_d	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000

The bottleneck for mean discharge time indicates that the reduction of the average service/waiting time of this step can lead to the largest decrease in the expected discharge time. Using the gamma approximation method, the improvements of mean discharge time are listed in Table V with $\delta_\tau = 0.1$. As one can see, a 10% reduction of τ_1^{RPH} can lead to the largest improvement in T_d, which is 18.48 min, making the "wait for order" the bottleneck of the discharge process here. More efforts should be focused on finding the factors or causes that can reduce this waiting time more significantly. In addition, the reductions of τ_1^{TR} and τ_9^{RPH} can also lead to substantial improvements in expected total discharge time, which are 1.12 and 1.08 min, respectively. It indicates that the transportation work and the "wait" after "contact physician for agreement" in the RPH workflow are also critical due to the large values of mean waiting times of these two steps.

Moreover, the reductions in CV of discharge time are listed in Table VI with $\delta_{\text{cv}} = 0.1$. Again, the reduction of cv_1^{RPH} leads to the largest improvement in CV_d, which is 0.0420, making the step of "wait for order" the bottleneck for variability of discharge process. The reduction of CV in transportation work leads to an improvement of 0.0035 in the CV of discharge time, while other reductions result in almost no changes.

C. What-If Analysis

In addition to mitigating bottlenecks, we next investigate how other parameters may impact discharge-time performance. Especially, we conduct what-if analysis with respect to the rates of unexpected discharge, discharging to aftercare facility, transportation need, and pharmacist intervention.

TABLE VII
WHAT-IF ANALYSIS WITH RESPECT TO UNEXPECTED DISCHARGE RATE

T_{given}	Unexpected discharge rate	2%	10%	20%	30%	40%	50%
300	DTP	0.6644	0.6644	0.6644	0.6644	0.6644	0.6644
	Percentage	-	0%	0%	0%	0%	0%
400	DTP	0.8310	0.8310	0.8310	0.8310	0.8310	0.8310
	Percentage	-	0%	0%	0%	0%	0%
500	DTP	0.9193	0.9193	0.9193	0.9193	0.9193	0.9193
	Percentage	-	0%	0%	0%	0%	0%

TABLE VIII
WHAT-IF ANALYSIS WITH RESPECT TO GO TO AFTERCARE FACILITY RATE

T_{given}	Discharging to aftercare facility rate	25%	30%	35%	40%	45%	50%
300	DTP	0.6644	0.6648	0.6652	0.6656	0.6660	0.6664
	Percentage	-	0.1%	0.1%	0.2%	0.2%	0.3%
400	DTP	0.8310	0.8312	0.8315	0.8317	0.8319	0.8321
	Percentage	-	0%	0.1%	0.1%	0.1%	0.1%
500	DTP	0.9193	0.9194	0.9195	0.9196	0.9197	0.9199
	Percentage	-	0%	0%	0%	0%	0.1%

TABLE IX
WHAT-IF ANALYSIS WITH RESPECT TO TRANSPORTATION RATE

T_{given}	Need transportation rate	14%	20%	25%	30%	35%	40%
300	DTP	0.6644	0.6529	0.6433	0.6338	0.6242	0.6146
	Percentage	-	-1.7%	-3.2%	-4.6%	-6.1%	7.5%
400	DTP	0.8310	0.8226	0.8157	0.8087	0.8017	0.7947
	Percentage	-	-1.0%	-1.8%	-2.7%	-3.5%	-4.4%
500	DTP	0.9193	0.9140	0.9095	0.9050	0.9005	0.8961
	Percentage	-	-0.6%	-1.1%	-1.6%	-2.0%	-2.5%

1) Unexpected Discharge: The unexpected discharge rate on the discharge date is typically low in practice since the SW and CM start their work in advance once the patient is admitted to hospital. As shown in Table VII, the DTPs only have insignificant changes when the rate of unexpected discharge increases from 2% to 50%. This is because the service/waiting times in the SW/CM workflow are much shorter than those in the RPH workflow, even if the discharges are unexpected.

2) Discharging to Aftercare Facility: If the patient is to be discharged to an aftercare facility, the service/waiting time will be longer in the SW/CM workflow since the SWs and CMs need to contact the aftercare facility, while the RPH workflow will have a shorter time since the pharmacist only needs to submit the information for discharge package instead of providing further education to the patient. As shown in Table VIII, the DTPs can be improved when the rate of discharging to aftercare facility increases. However, the improvements are still insignificant or moderate. This is because the probability that SW/CM needs to contact aftercare facility for unexpected discharge is quite small though the contacting time is longer than the pharmacist's education time.

3) Transportation Need: The expected preparation time for transportation work is 233 min for 14% of all patients at UW Hospital. As shown in Table IX, the DTPs will decrease significantly when the rate of transportation increases from 14% to 40%. When the rate of transportation need is 40%, the decrease in DTP will be 7.5% for T_{given} to be 300.

4) RPH Intervention: The medication reconciliation is applied to all patients to identify potential discrepancies, such as medical issues, questions, or errors and adverse drug events, in the prescription medication orders. If a discrepancy is found, pharmacist intervention is needed to change the order. The order change rate in RPH intervention is high at UW Hospital, which is estimated around 60%. As shown in Table X, the DTPs will be improved moderately if the intervention rate decreases from 60% to 10%. This result indicates that improving coordination and consensus between physicians and pharmacists in prescription orders to reduce RPH intervention rate could help reduce discharge time.

D. Discussion

Based on the above-mentioned analyses, it can be seen that the waiting time for physician's order is the main bottleneck of the discharge process. In addition, the transportation time will become an issue if the rate requiring transportation is increased (although less likely). Thus, to reduce discharge time, the first

TABLE X
WHAT-IF ANALYSIS WITH RESPECT TO RPH INTERVENTION RATE

T_{given}	RPH intervention rate	60%	50%	40%	30%	20%	10%
300	DTP	0.6644	0.6693	0.6743	0.6792	0.6841	0.6890
	Percentage	-	0.7%	1.5%	2.2%	3.0%	3.7%
400	DTP	0.8310	0.8337	0.8364	0.8391	0.8417	0.8444
	Percentage	-	0.3%	0.6%	1.0%	1.3%	1.6%
500	DTP	0.9193	0.9206	0.9219	0.9233	0.9246	0.9259
	Percentage	-	0.1%	0.3%	0.4%	0.6%	0.7%

priority is to minimize the waiting time for physician's order. In other words, the physician is dominating the process to make the final decision, which can be viewed as the captain of the ship in the discharge process. From this perspective, the whole discharge process can be shortened significantly if the physician's order can be prescribed at an earlier time, given that the patient condition is ready for discharge. In some cases, the long waiting time for physician's order is due to waiting for lab results to confirm the patient's conditions. Thus, it will be important to investigate how to reduce lab turnaround time for the discharging patients. Overall, it would be ideal if the provider (MD/APP) could write an "intent to discharge tomorrow" order so that some of the workflows can be started. Some providers are hesitant to do this because of the possibility of the patient's condition changing, labs coming back abnormal, and so on.

Although the RPH workflow is still the most critical subprocess in the whole discharge process, the pharmacist is not the constraint or bottleneck in the UW Hospital's discharge process. Currently, the pharmacist may have many interventions during the discharge period. However, reducing the pharmacist's interventions can only lead to moderate improvement, and such interventions are important since they can effectively reduce medication errors and adverse drug events [36]. Therefore, making the pharmacist focus on the complex duties and keeping rigorous checks to eliminate medication errors and adverse drug events are critically needed. If the physician's order can be prescribed at an earlier time and the SW/CM can set up the transportation earlier, then the process efficiency will be more sensitive to RPH workflow. Such dedications will be more important.

In this paper, the discharge process is not critical to the SW/CM workflow as many activities have been finished before the discharge order. Thus, there is no need to hire more SW or CM from the discharge improvement perspective. Note that this assumes that the place of discharge (where the patient is going to) can be easily arranged. Sometimes it is very difficult to place patients due to their medical complexity or because they do not have insurance and so on. Such issues can consume the SW/CM time.

Moreover, there exist many complicated delay factors, such as medical consultation or investigation, various disruptions, and variability in patient's condition and patient's mood, in the discharge process. Such factors could be medical or nonmedical-related, internal or external reasons, or due to psychological issues. Thus, coordination and communication among the team members in the discharge process play a key role. The physician is the "captain of the ship" since the discharge order is needed to "get the ball rolling." The nurse and the SW/CM are the ones who begin the process and keep it rolling. The nurse involvements to coordinate with physicians, therapists, lab technologist, pharmacists, and CMs are crucial to achieve timely and high-quality discharges.

VII. CONCLUSION

In this paper, an analytical model is introduced to study the hospital discharge process. By systematically characterizing the discharge process and using an aggregation approach and gamma approximation, the mean discharge time, the variability, and the discharge-time performance can be evaluated with high accuracy. It is shown that such performances are practically independent of the distribution of service/waiting time at each step in the discharge process but primarily depends on their mean and CV. In addition, using the approximation method, the monotonicity properties are investigated. Based on the data collected at the UW Hospital, the bottleneck analysis is carried out to identify the most impeding step in the process, which turns out to be the waiting time for physician's discharge order.

Such a model provides a quantitative tool for hospital management to study and improve the discharge process. It is not only applicable to UW Hospital but also useful for analyzing discharge processes in other hospitals. Moreover, the methodology can be applied to production, product development, and other engineering fields with multiple concurrent processes. To extend this study, a more in-depth analysis of the discharge process can be carried out. Particularly, the following topics are of interest as follows:

1) assumption of phase-type or other distributions (such as Pearson distribution) for service (or waiting) time at each step;
2) communication and coordination between multiple care providers during discharge;
3) multiple tasks for providers, which relates to availability and resulting waiting time of the providers;
4) patients' and families' involvement in discharge;
5) different workflows, configurations, and protocols or guidelines for discharge;
6) alliance with predictive models of patient conditions;
7) integration with analysis of admission, readmission, and patient transitions within hospitals.

APPENDIX
PROOFS

A. Proof of Proposition 1

It has been shown in [21] that if $\{X_i, i = 1, \ldots, n\}$ are independently distributed gamma random variables with mean τ_i and standard deviation σ_i, then

$$G(T_{\text{given}}) = \prod_{i=1}^{n} \left(\frac{\beta_{\min}}{\beta_i}\right)^{\eta_i} \sum_{k=0}^{\infty} \frac{\delta_k \gamma(\rho + k, T_{\text{given}}/\beta_{\min})}{\Gamma(\rho + k)} \tag{A.1}$$

where $\beta_{\min}$, β_i, η_i, δ_k, ρ, $\gamma(\cdot)$, and $\Gamma(\cdot)$ are defined in (13). Using this result, the number of steps in serial process i can be defined by n_i^l. Then, DTP_i^l of procedure i of workflow l for time interval T_{given} can be evaluated by $G_i^l(T_{\text{given}})$. Then, DTP^l can be calculated using the weighted sum of the cdfs of all the pathways in workflow l, i.e.,

$$\text{DTP}^l = \frac{\sum_i p_i^l G_i^l(T_{\text{given}})}{\sum_i p_i^l}.$$

Taking the maximum, we obtain the overall discharge-time performance of the whole process.

B. Proof of Proposition 2

For the mean of waiting time τ_i^l, it is obvious that

$$\frac{\partial G_j^l(T_{\text{given}})}{\partial \tau_i^l} < 0$$

when route j of workflow l includes step τ_i^l; otherwise, $G_j^l(T_{\text{given}})$ does not change with τ_i^l.

In addition, for workflow $k \neq l$, $k \in \{\text{SW}, \text{TR}, \text{RPH}\}$, we have

$$\frac{\partial G_j^k(T_{\text{given}})}{\partial \tau_i^l} = 0.$$

That is to say, the discharge-time performance of a workflow decreases with respect to the mean of waiting time of the step from the same workflow and does not change with the one from a different workflow

$$\frac{\partial \text{DTP}^l}{\partial \tau_i^l} < 0 \tag{A.2}$$

$$\frac{\partial \text{DTP}^k}{\partial \tau_i^l} = 0, \text{ for } k \neq l. \tag{A.3}$$

Thus, from (3), we obtain

$$\frac{\partial \text{DTP}}{\partial \tau_i^l} = \frac{\partial \prod_{k=\text{SW,TR,RPH}} \text{DTP}^k}{\partial \tau_i^l} < 0.$$

C. Proof of Proposition 3

From Proposition 2, by (A.2) and (A.3), we obtain

$$\begin{aligned}\frac{\partial T_d}{\partial \tau_i^l} &= \frac{\partial \int_0^\infty (1 - \text{DTP}) dT_{\text{given}}}{\partial \tau_i^l} \\ &= \frac{\partial \int_0^\infty (1 - \prod_{k=\text{SW,TR,RPH}} \text{DTP}^k) dT_{\text{given}}}{\partial \tau_i^l} \\ &> 0.\end{aligned}$$

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